Sampling-Based Robot Motion Planning

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Motion planning: classical setting

Go from Start to Goal without collisions and while respecting all robot constraints.

Motion Planning

- Is done in a continuous world and with constrained motions.
- Needs to know robot and world geometry.
- Needs to know robot and world physics.
- Must be accurate and predictive to work in practice.

Some notes:

- More powerful motion planning simplifies the task planner.
- More accurate motion planning simplifies motion execution.
- Motion planning is limited by model accuracy and complexity.

Motion planner is part of a replanning loop

Bekris et al.

Motion planning is hard

Problem	Complexity
Sofa Mover (3 DOF)	$O(n^{2+\varepsilon})$ not implemented
Piano Mover (6 DOF)	Polynomial – no practical algorithm known
n Disks in the Plane	NP-hard
n Link Planar Chain	PSPACE-Complete
Generalized Mover	PSPACE-Complete
Shortest Path for a Point in 3D	NP-hard
Curvature Constrained Point in 2D	NP-hard
Simplified Coulomb Friction	Undecidable

Exact, approximate, and heuristic methods

Method	Advantage	Disadvantage
Exact	theoretically insightful	impractical
Cell Decomposition	easy	does not scale
Control-Based	online, very robust	requires good trajectory
Potential Fields	online, easy	slow or fail
Sampling-based	fast and effective	cannot recognize impossible query

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Outline of this talk

- Basic concepts and definitions.
- Examples of sampling-based planners:
  - Roadmap planner
  - Tree-based planner
- Underlying key components.
- OMPL and future challenges in motion planning.

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Basic concepts and definitions

- Workspace
- Robot
- State space
- Path
- Planning Instance/Simple Setup
- Query/Problem

Workspace

- The workspace is the environment that the robot operates in.
- The boundary of the workspace determines the obstacles.

Robot

- The robot is defined by:
  - Geometry
  - Parameters or Degrees of Freedom (DOF)
    - Different settings for the parameters embed the geometry in different ways into the workspace.

State space

- The parameter space for the robot is called the state space S.

- A point in this space is a state.

Free state space

- A state is free if the corresponding embedding of the robot's geometry lies in the workspace.

- The subspace of free configurations is free state space S_{free}.

- S_{free} can be very complex even for seemingly simple systems.

- This complexity is the main difficulty in motion planning.

Paths

- A path is continuous mapping in C

$$\pi : [0, L] \rightarrow S_{free}$$

- L is the length of the path.

- The path is collision-free if for all t

$$\pi(t) \in S_{free}$$

Planning instance/Simple setup

- A planning instance consists of:
  - Robot (S-space and embedding).
  - Workspace.
  - Constraints.

Query/Problem definition

- A problem or query is
  - Given two states, q_0 and q_f.

PROBLEM:

Determine if there is a collision-free path between q_0 and q_f.

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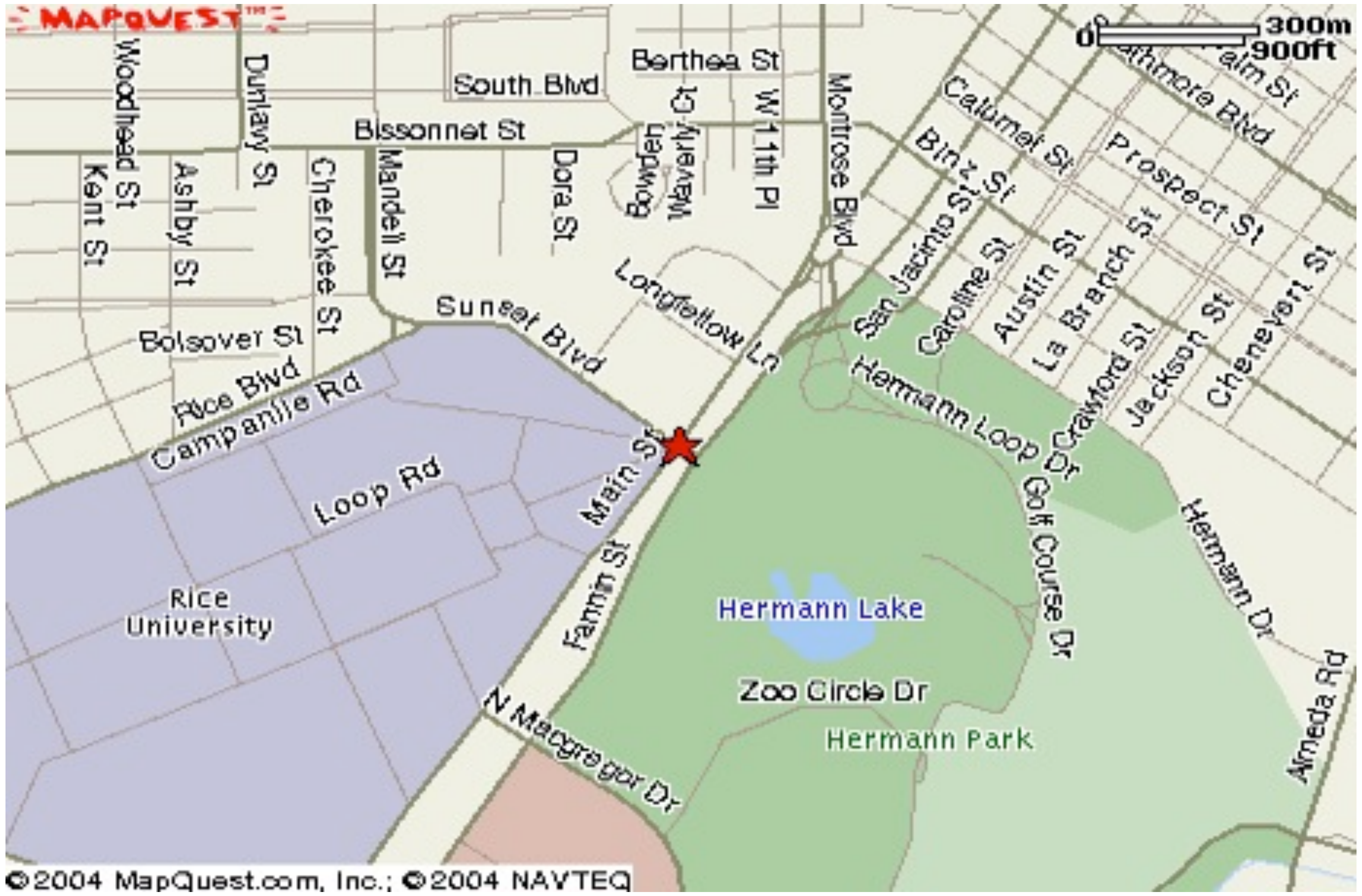
Probabilistic Roadmap Planner (PRM)

Kavraki, Svestka, Overmars and Latombe 96

PRM

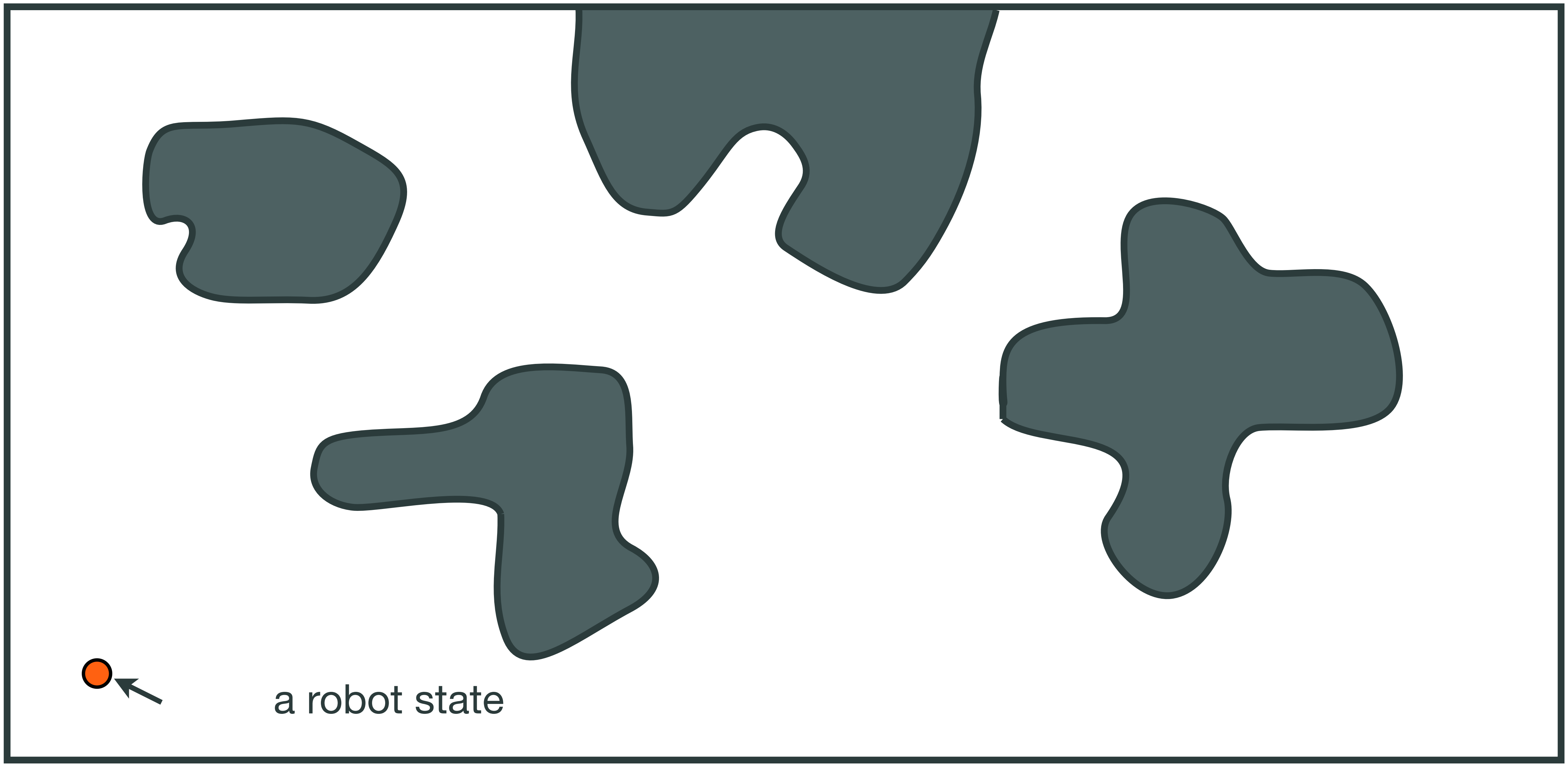
- Uses random sampling.
- Uses simple local planner.
- Builds a roadmap of the state space.



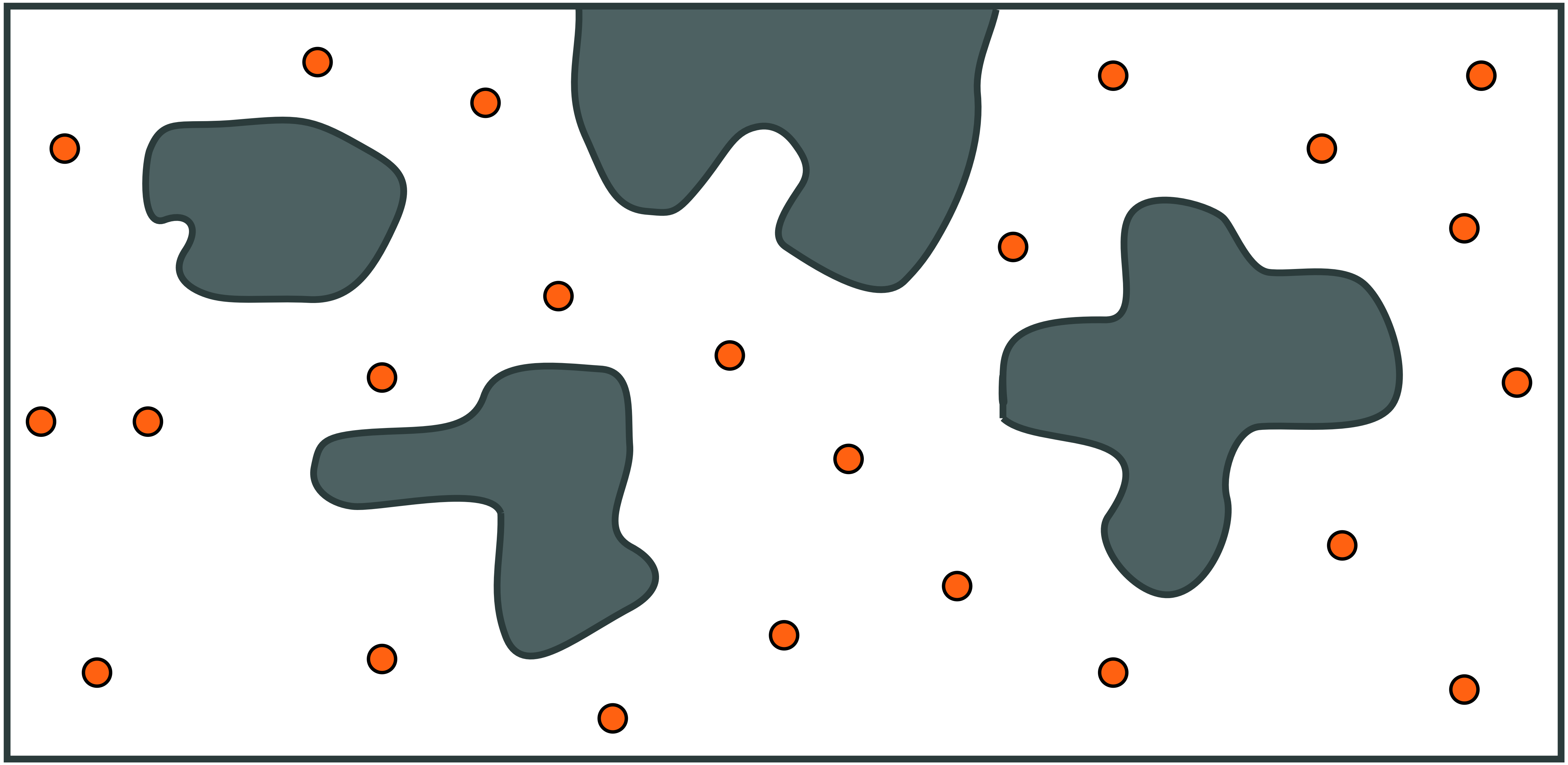
PRM

- Illustrate with an easy planning instance/problem set up.
    - Robot is a point in 2D.
    - Robot moves freely.
    - Simple example used for illustration only.
- Isolate primitive techniques.
- Generalize.

Point robot in 2-D

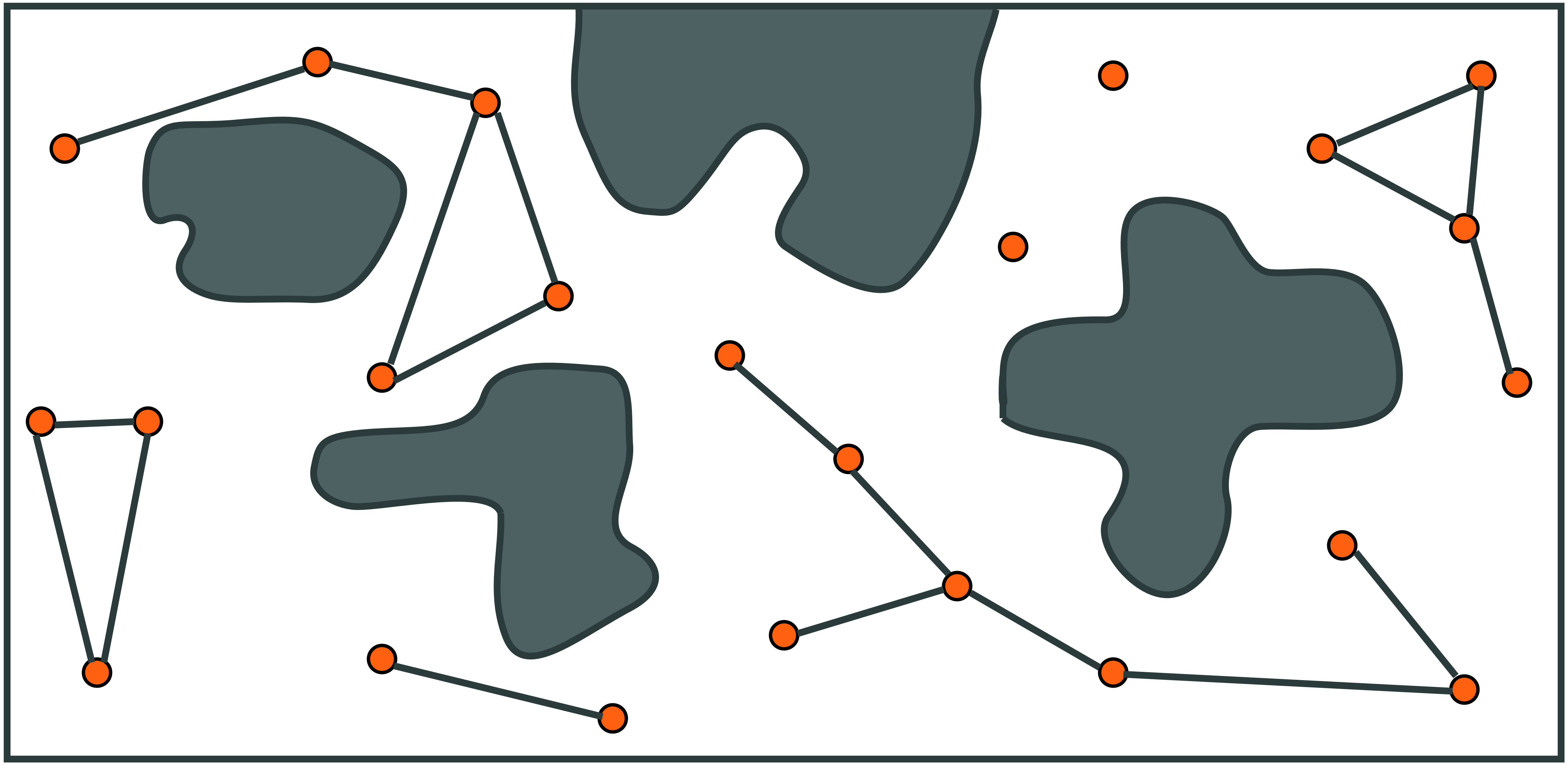


Operation of PRM



: **nodes,** random states

Operation of PRM



:edges, paths computed by local planner

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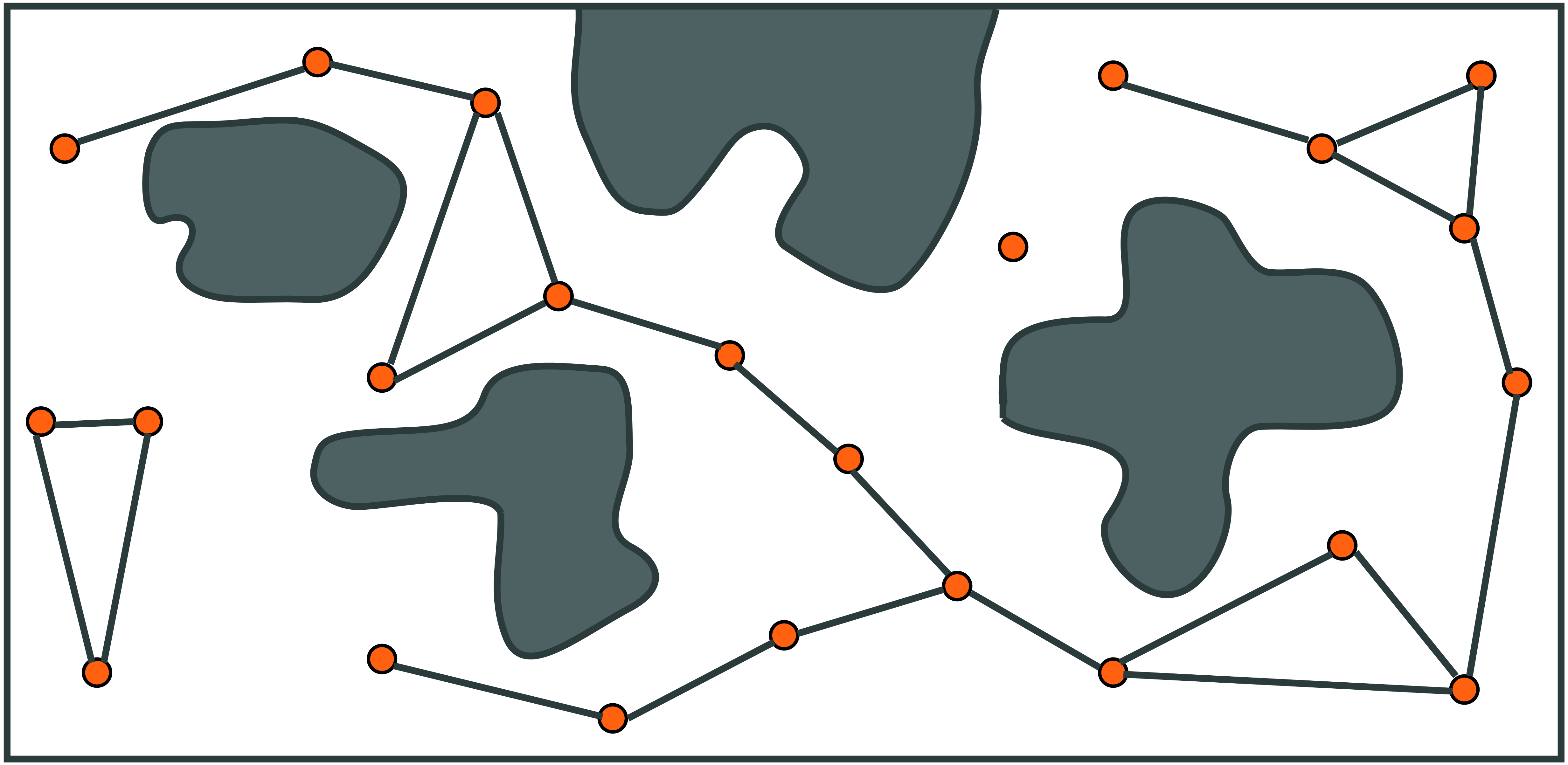

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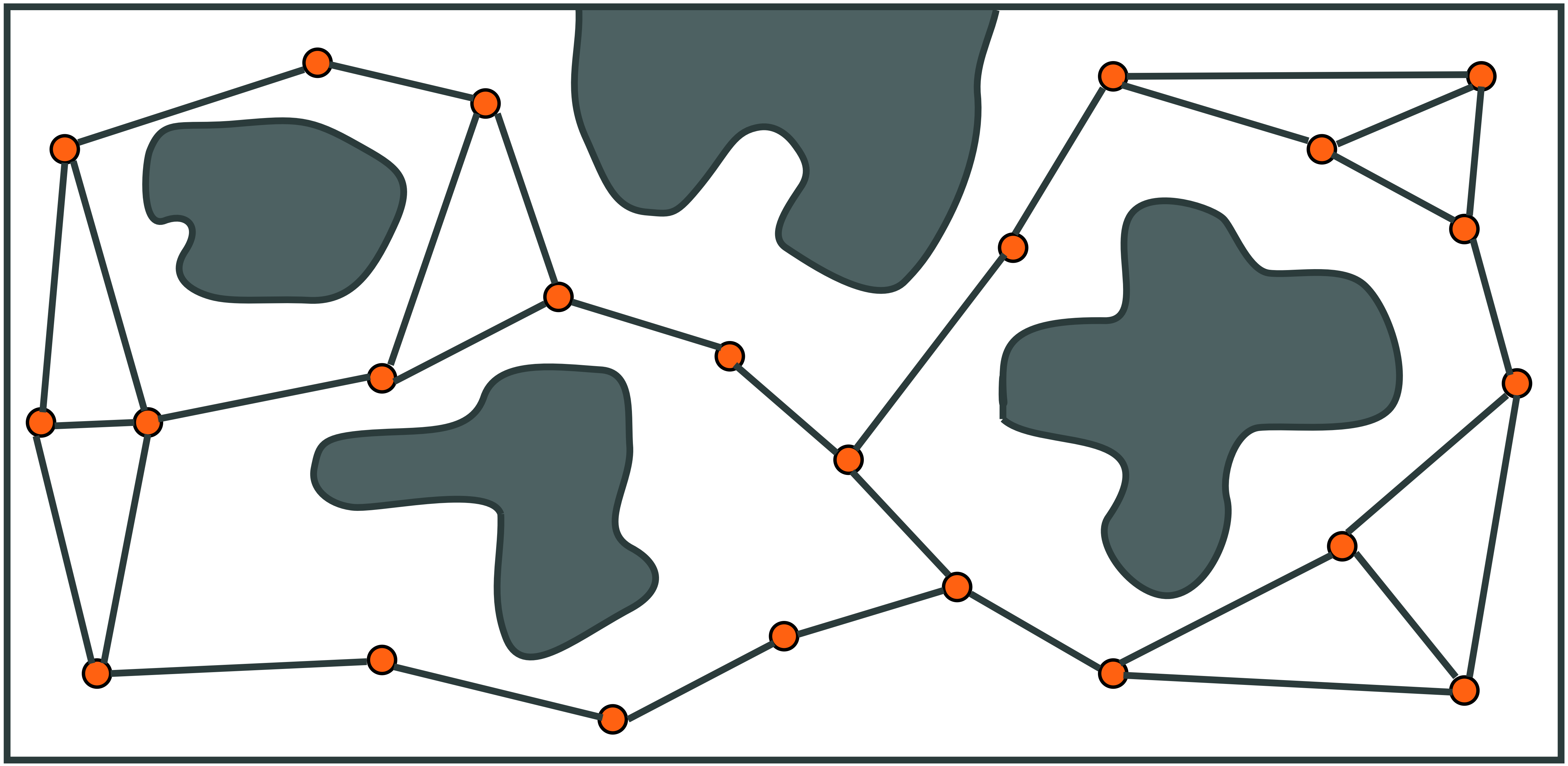

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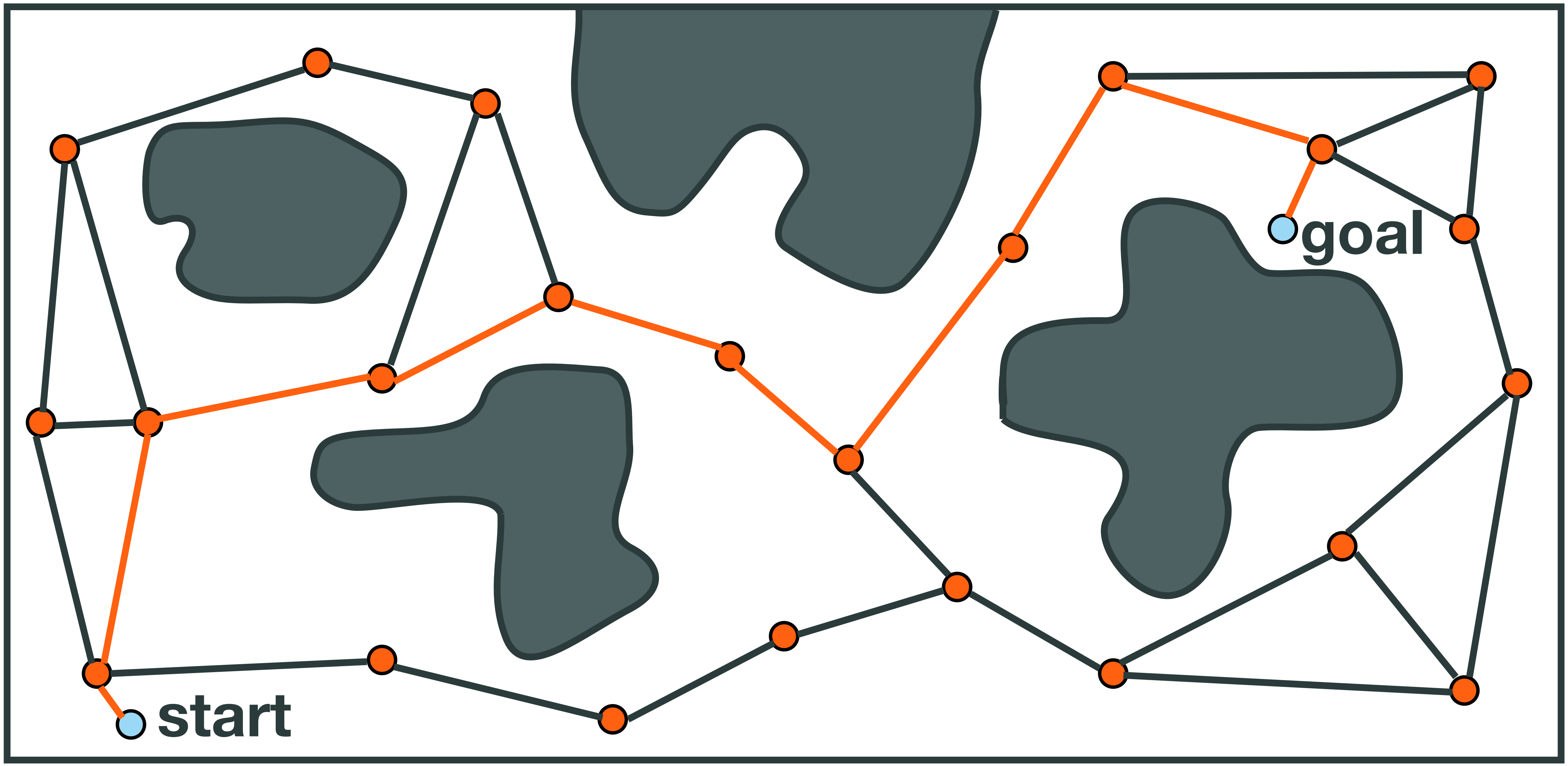
Queries

- Given a roadmap G and query q_0, q_f
  - Connect q_0 and q_f to G.
  - Check to see if there is a path in G.

Answering Queries



plan a path:

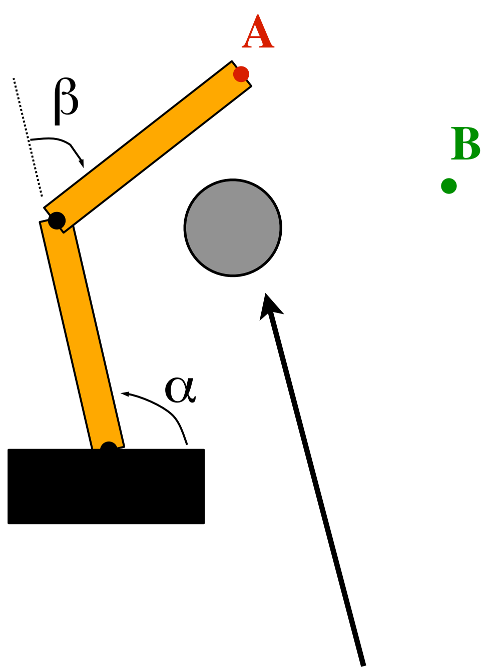
1. connect start & goal to roadmap
2. perform graph search

Primitive Techniques

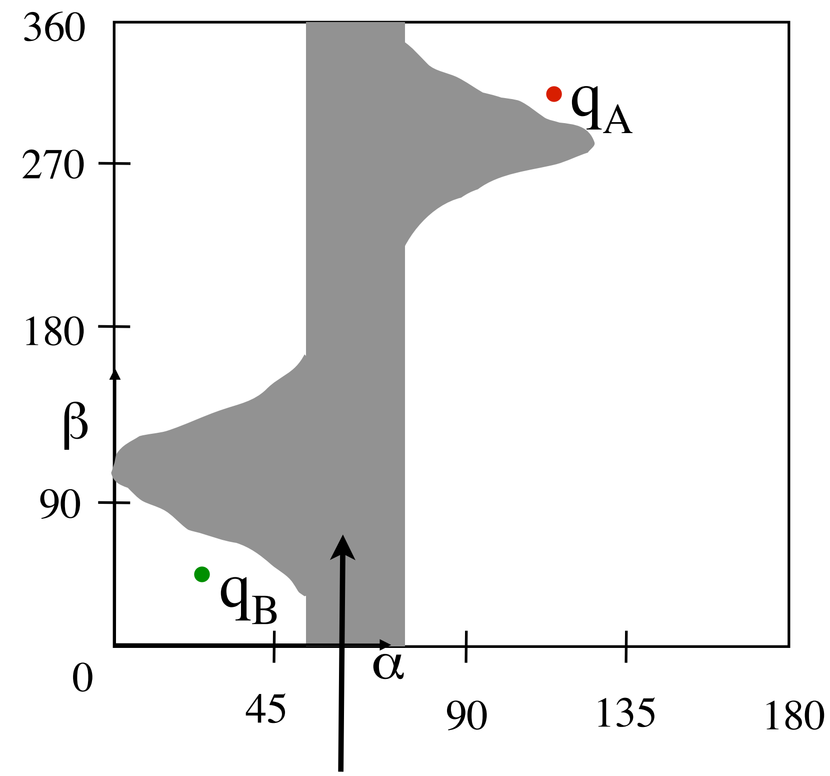
- **Select Sample:** (in the example) Uniform sampling to get milestones.
- **Connect:** (in the example) Local planner uses “straight lines.”
- **Store in some data structure:** (in the example) A graph.
    - A roadmap is finite graph G=(V,E)
    - V is a subset of S_{free}.
    - (s_1,s_2) in E implies that the local planner found a path.

Why use sampling?

- S_{free} is impractical to represent explicitly.



Obstacle in workspace



Obstacle in state space

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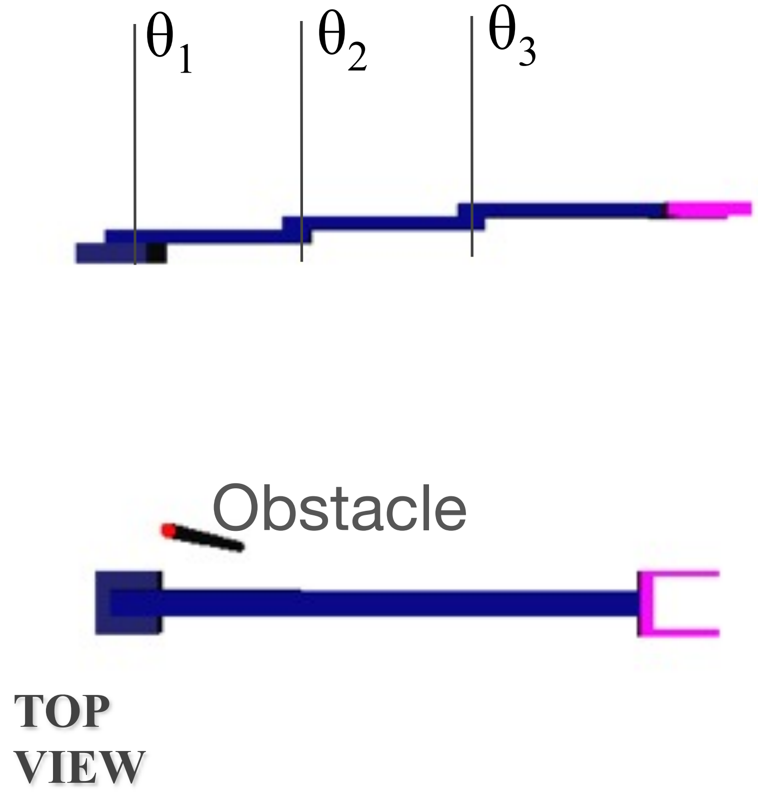

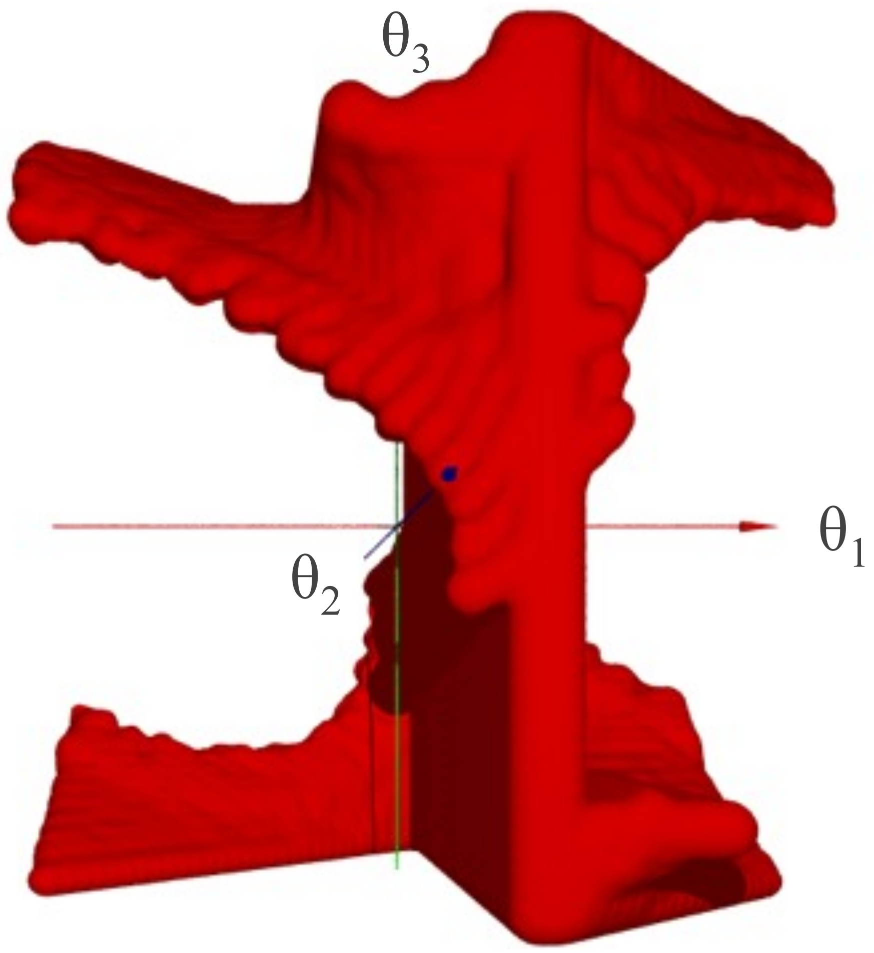

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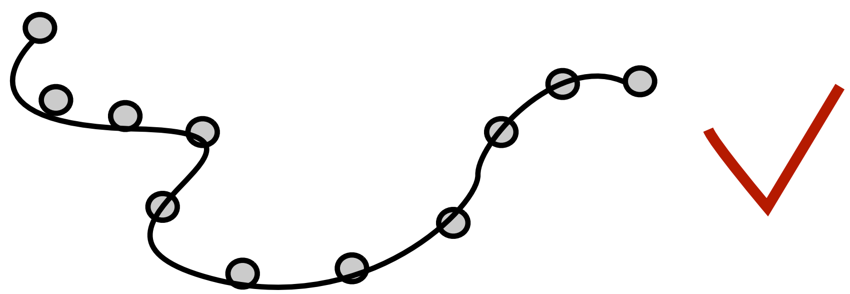
Why use sampling?

- S_{free} is impractical to represent explicitly.
- Sampling can be very efficient.
- Resulting data structure can be very compact.

Connecting samples

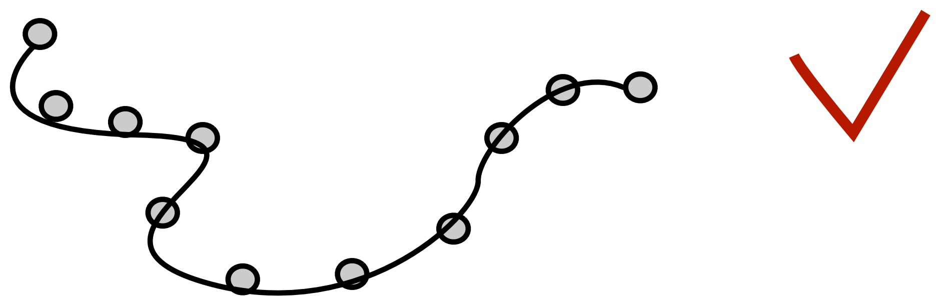
- An example of a simple planner:
  - Computes the straight line path between q1, q2.
  - Checks to see if it is valid.
  - If so, returns SUCCESS and the path.
  - Otherwise, returns FAIL.



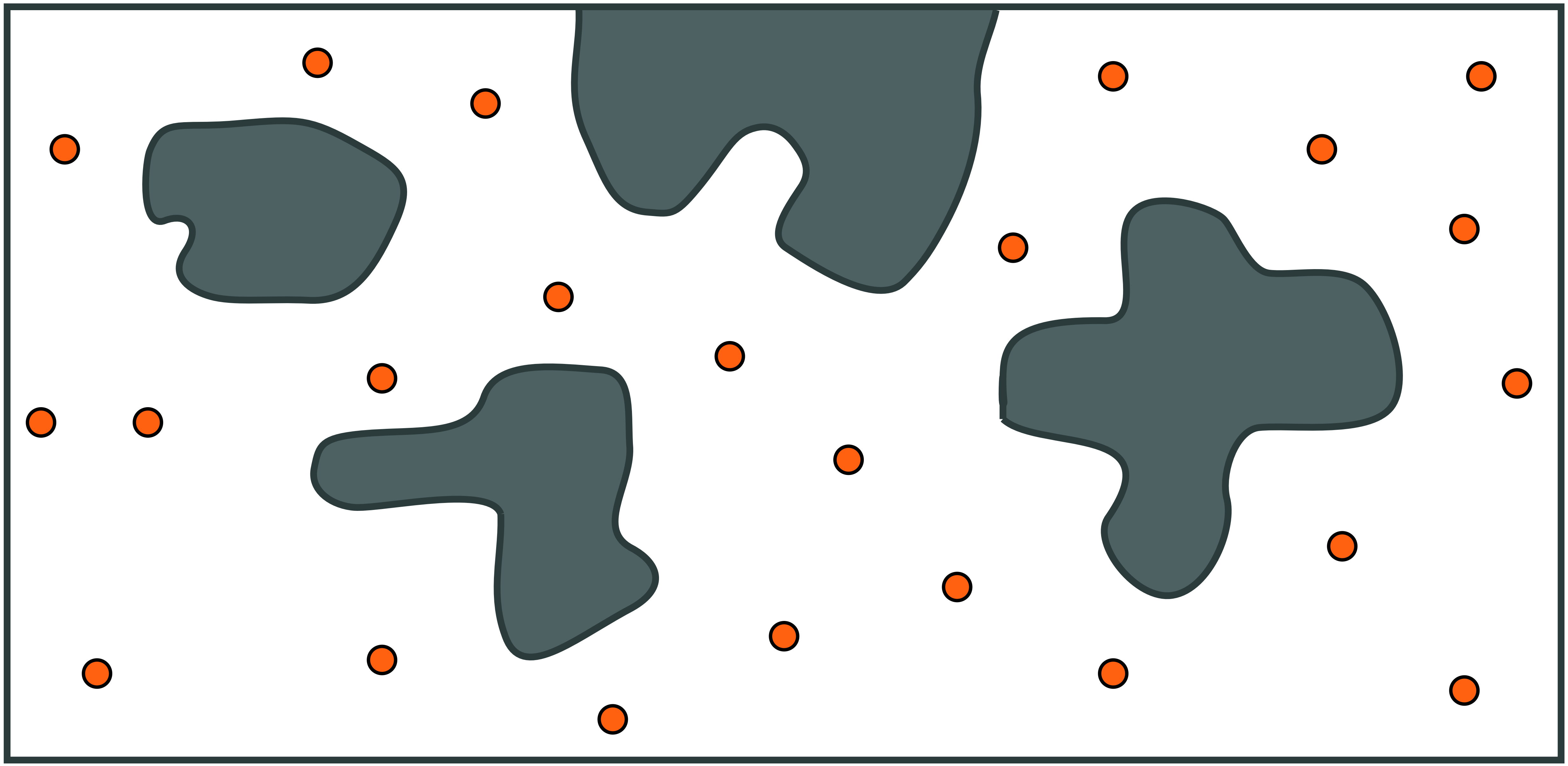
May fail often

State validity checker

- For states
    - Use e.g., collision checking, check any bounds
- For paths
    - State validation along a path is done by recursive refinement.
    - Bounds on clearance are combined with bounds on motion to cover the path with open balls or find a collision.



Operation of PRM



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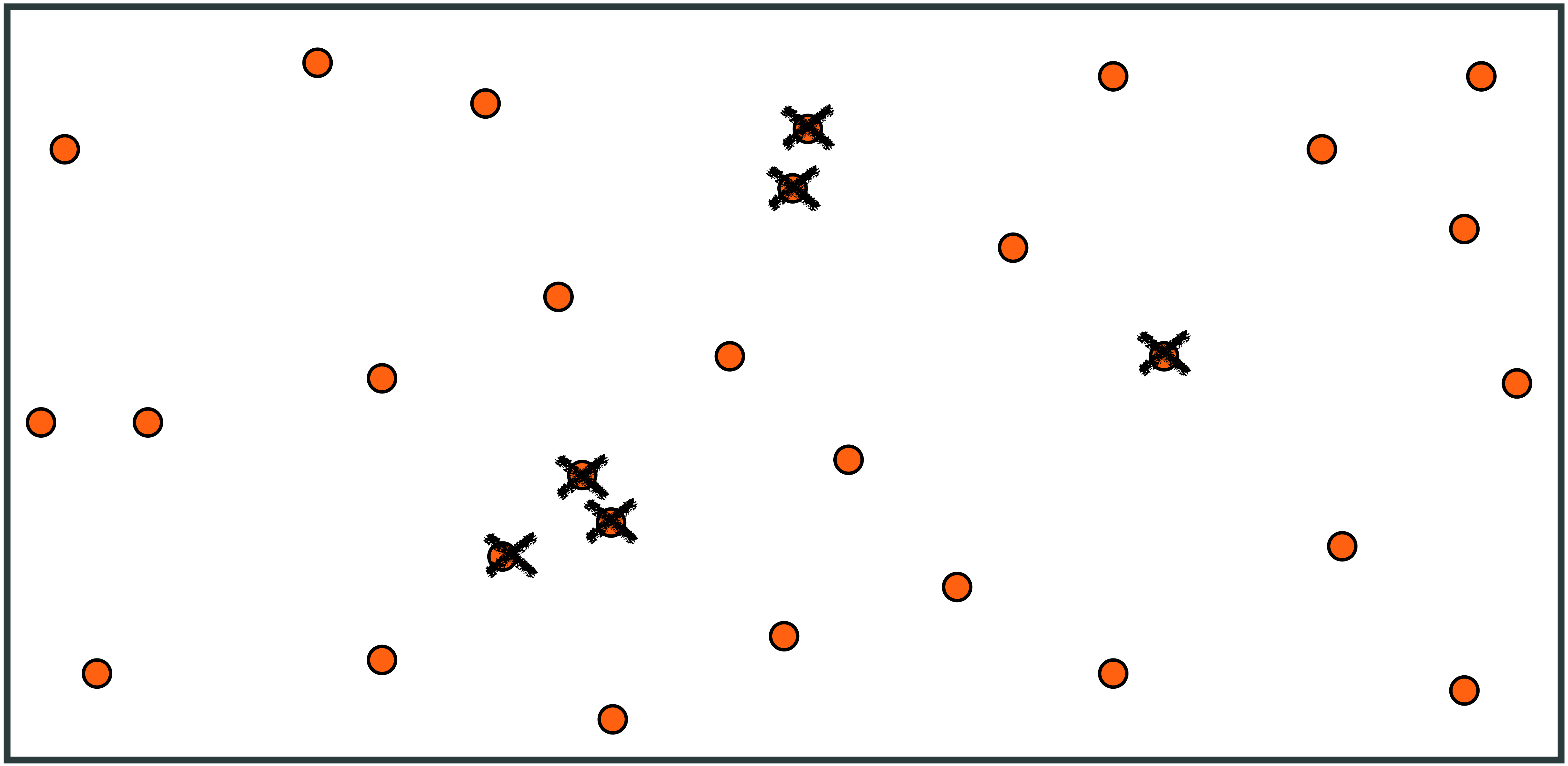

: states

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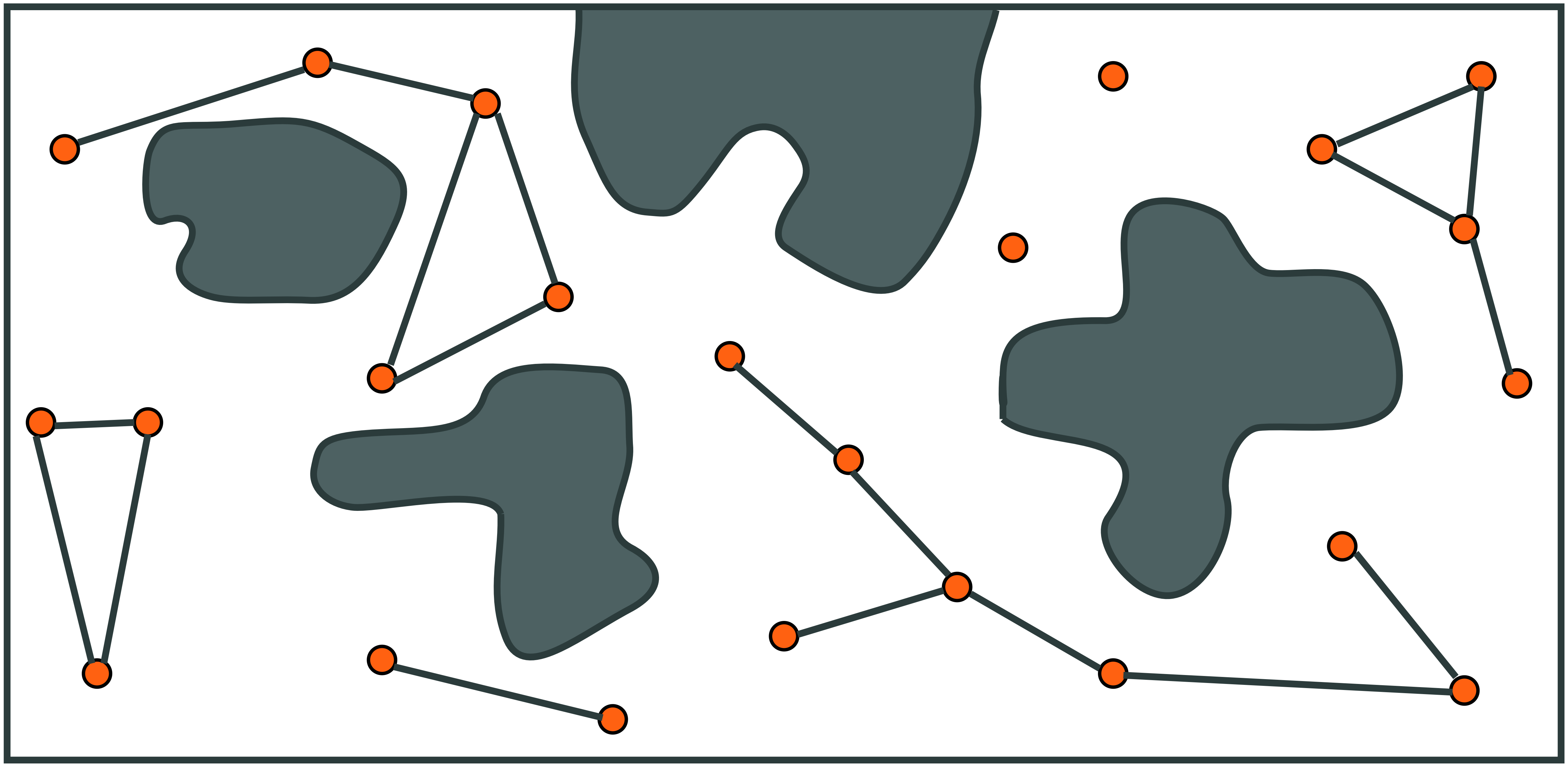

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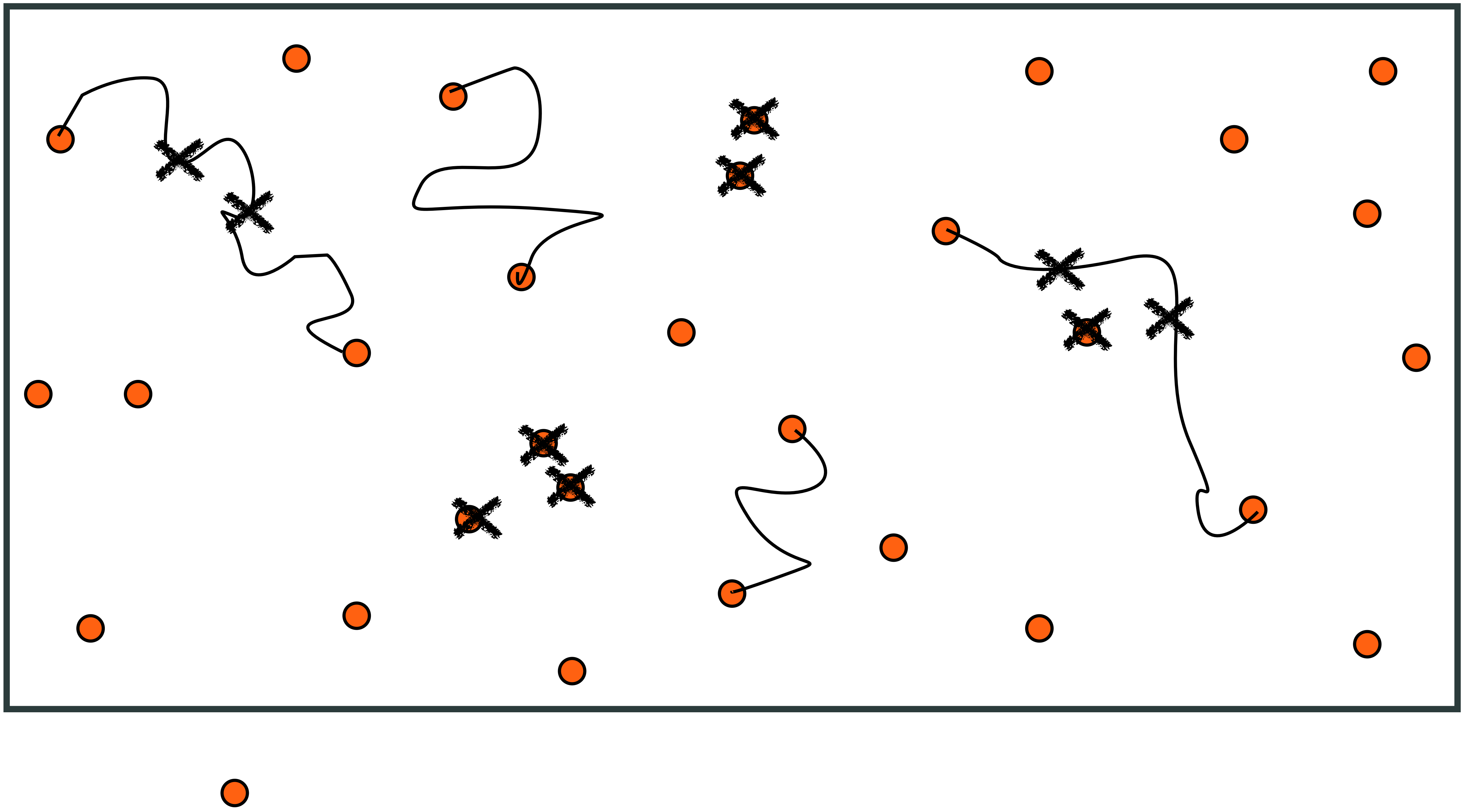

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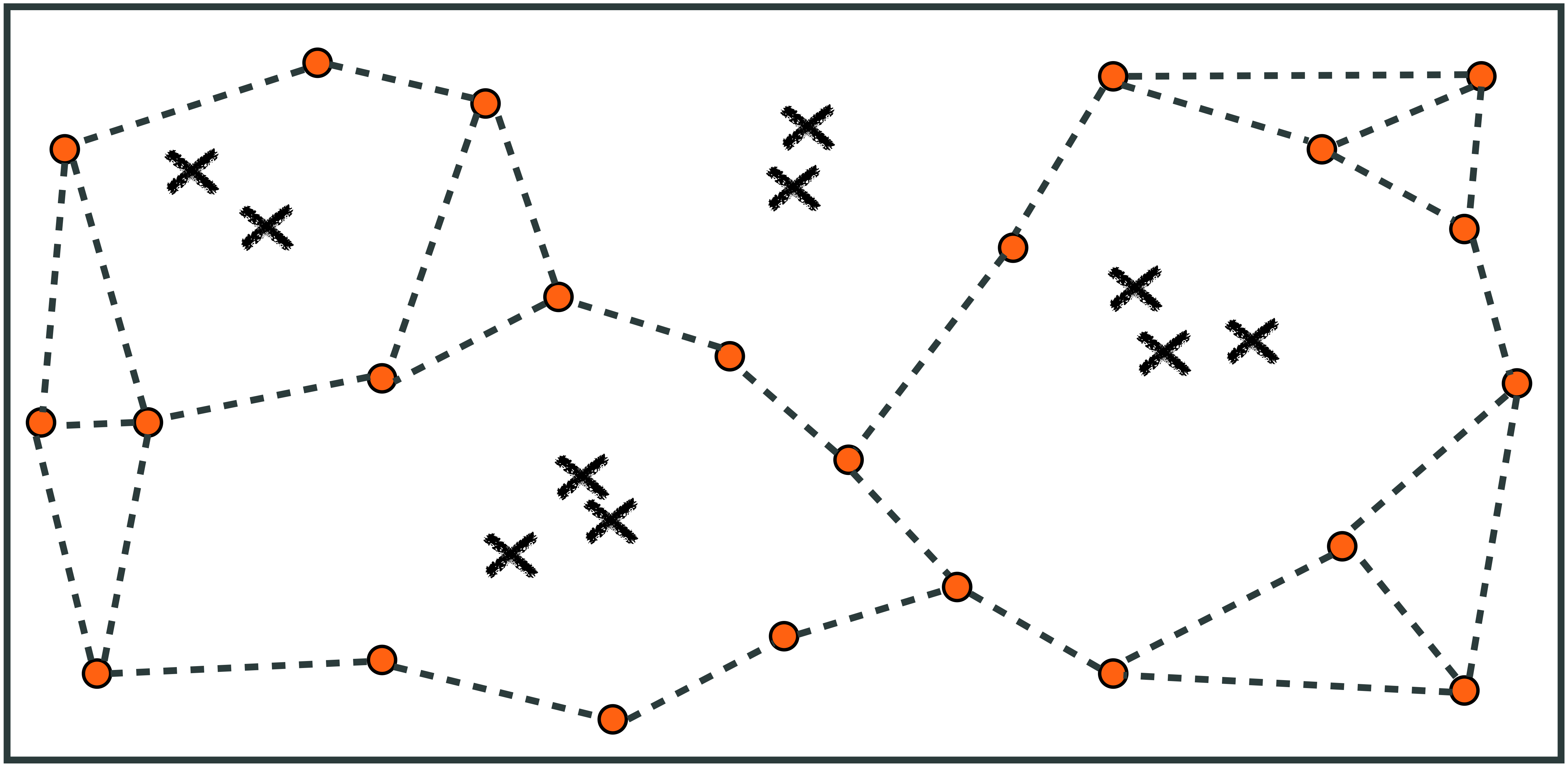
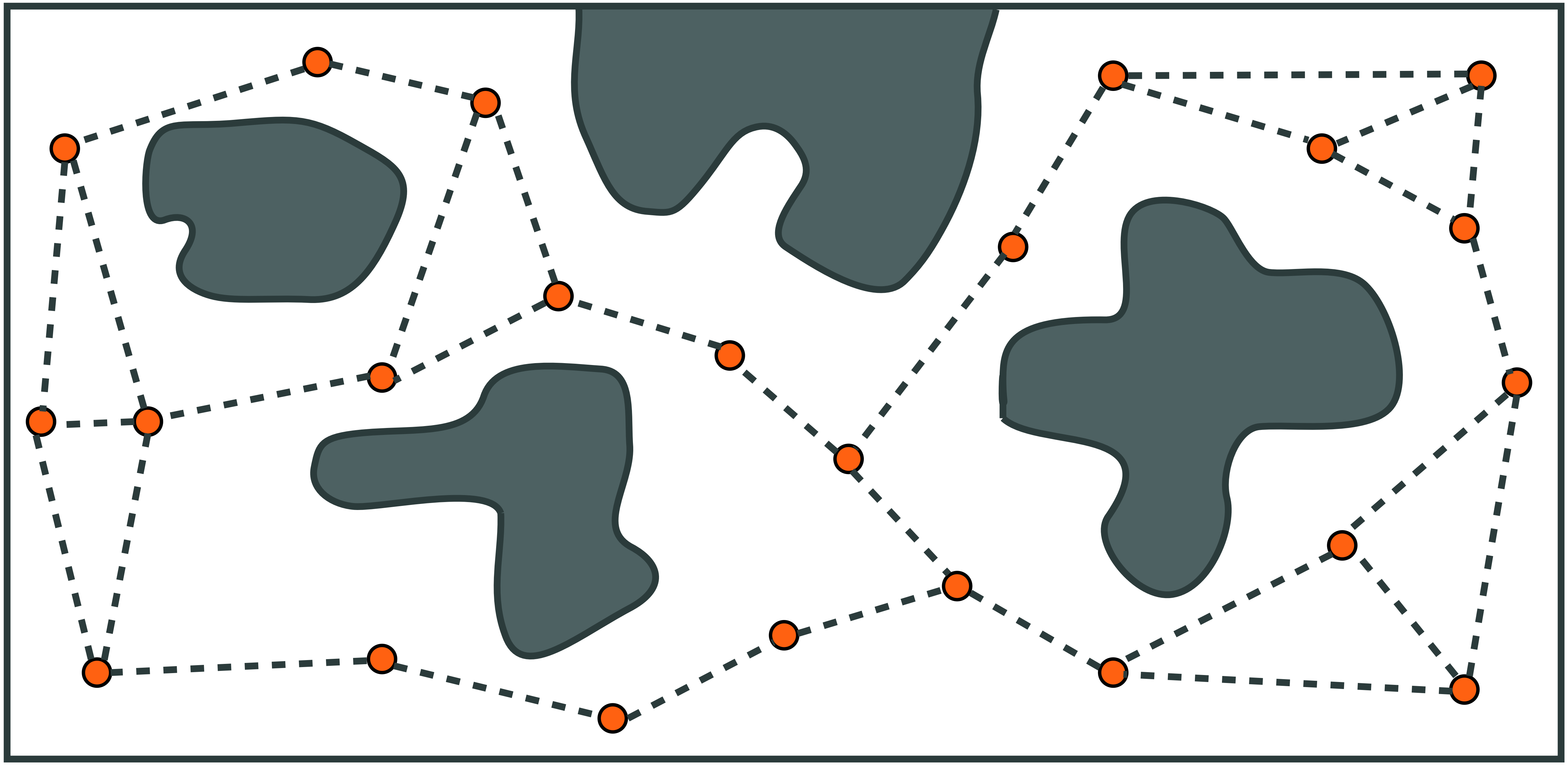


- - - - feasible path computed by local planner

Operation of PRM



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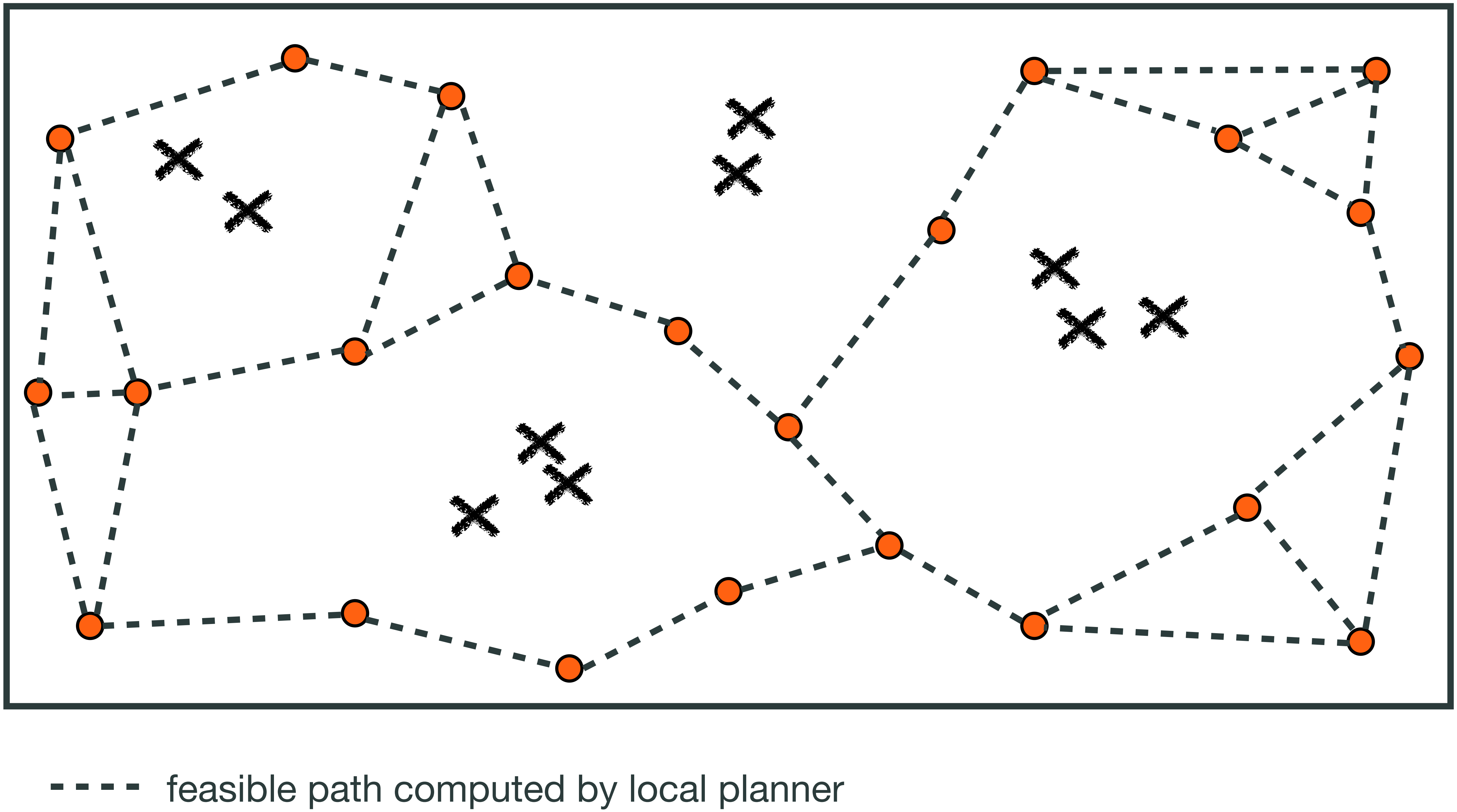
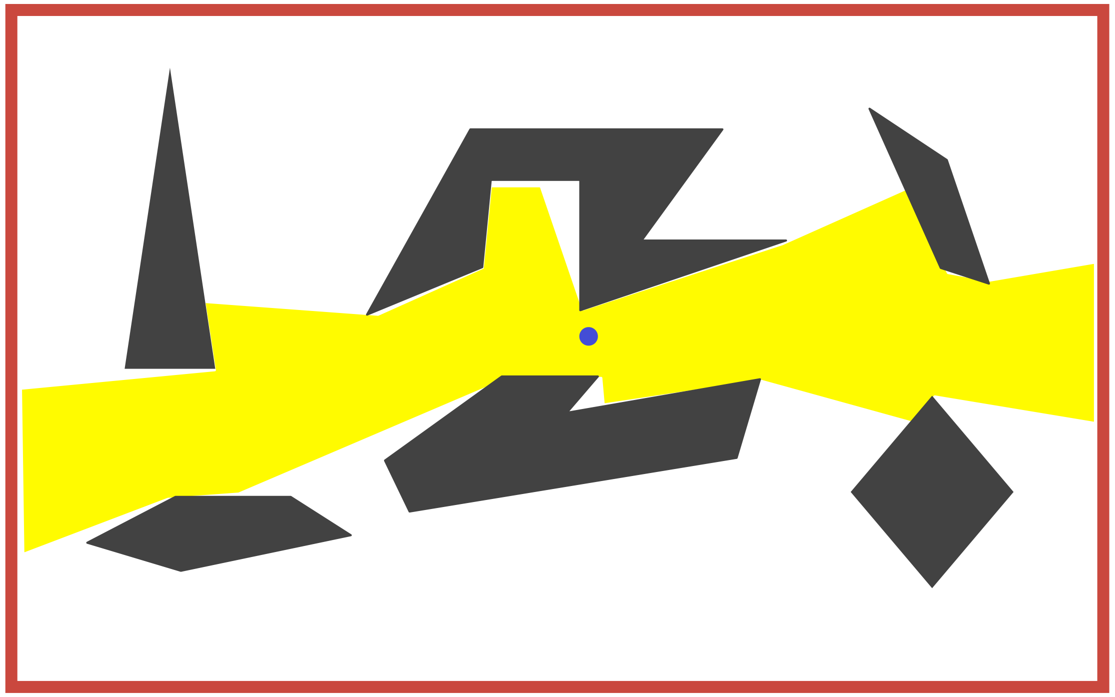


Completeness of PRM

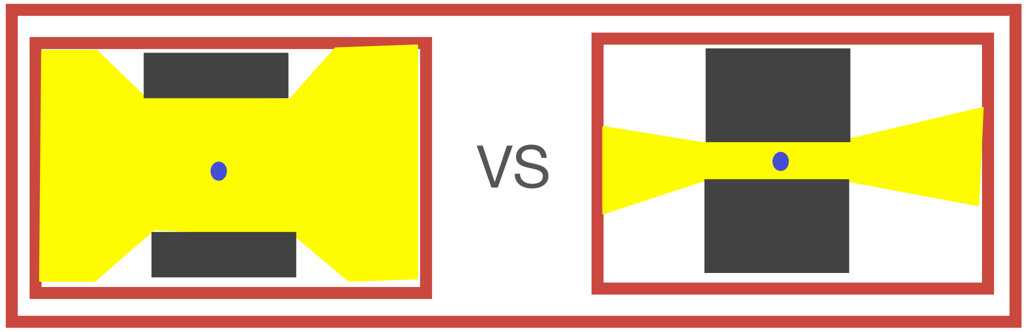
- If no path exists, then PRM cannot find the path.
- But… if a path exists, it is possible PRM fails to find it.
- PRM is not complete but instead is probabilistically complete.

Theoretical Analysis of PRM (1/2)

[Kavraki et al 96, 98, 00, 03, 07]



ε-goodness property



- Tradeoff: planner may fail with probability α
- Number of nodes/states:

$$N \approx \frac{1}{\epsilon}[log(\frac{1}{\epsilon}) + log(\frac{4}{\alpha})]$$

- Important: Performance related to properties of the space

Theoretical Analysis of PRM (2/2)

- We sacrifice completeness for speed
- Probabilistic completeness
- Novel analysis and performance guarantees

$$Pr(failure) = f(e^{-cN})$$

- How much can the assumptions be relaxed?

Primitive techniques

Primitives

- **Select Sample:** Uniform sampling is general but not the most efficient.
  - Optimal selection remains elusive.
- **Connect:** Connect all to all is general but not efficient.
  - Neighbors
  - Notion of “straight line” or other local plan needs to be adapted.
- **Store efficiently**

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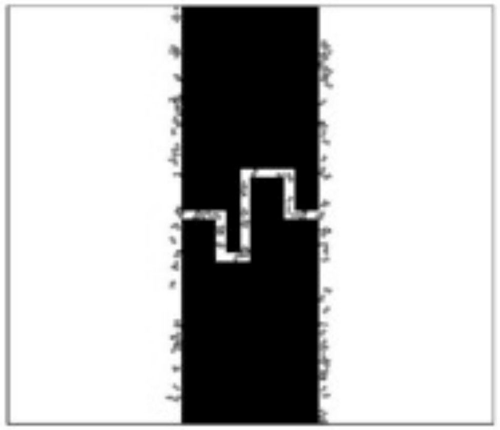
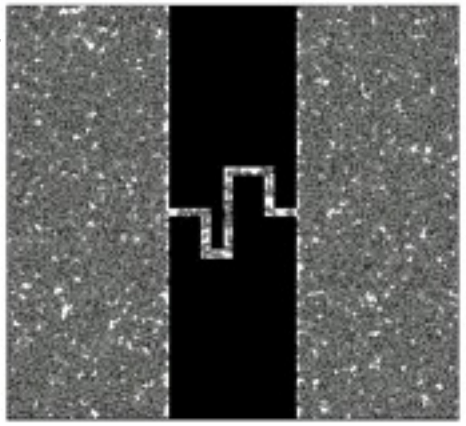
Several sampling strategies



VS



- Gaussian sampling [Overmars et al]:
    - Places samples close to objects.
    - Distribution is Gaussian around the obstacle boundary.
- Medical-axis sampling [Amato et al].
- Bridge Test sampling for narrow corridors [Hsu et al].
- Quasi-Random sampling [LaValle et al].
- Selective sampling [Kavraki el al].

Recent study confirmed it is one of the most critical parts of the planner [Hsu, Latombe 1998].

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Several connection strategies

- Nearness: Try to connect each configuration to a constant number of "nearby" configurations.
  - nearest neighbors by kd-trees, k-NN, k-ANN
  - random neighbors may be helpful
- Component technique: Only test edges which reduce the number of connected components in the roadmap.

Svestka, Overmars 96

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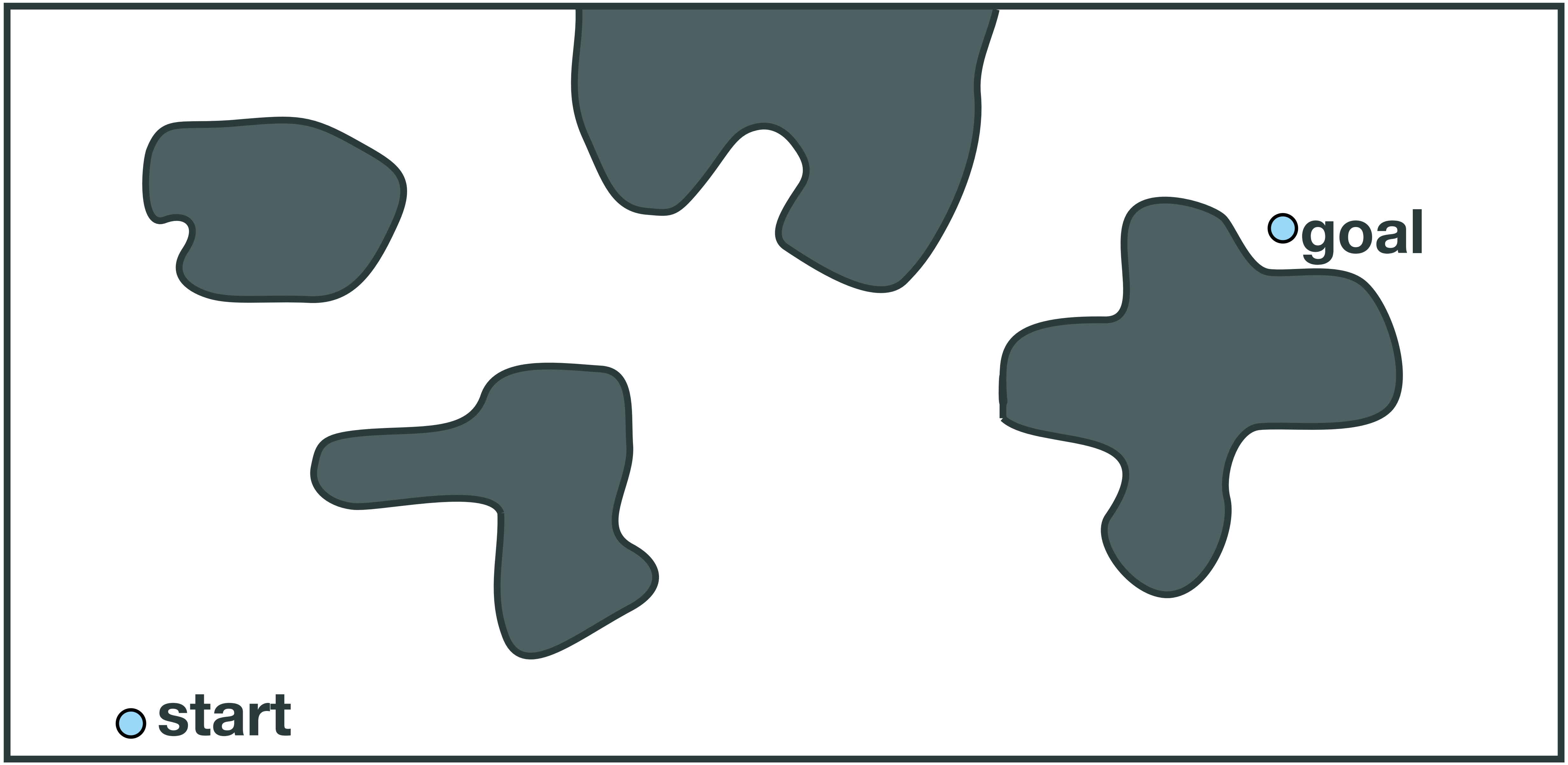
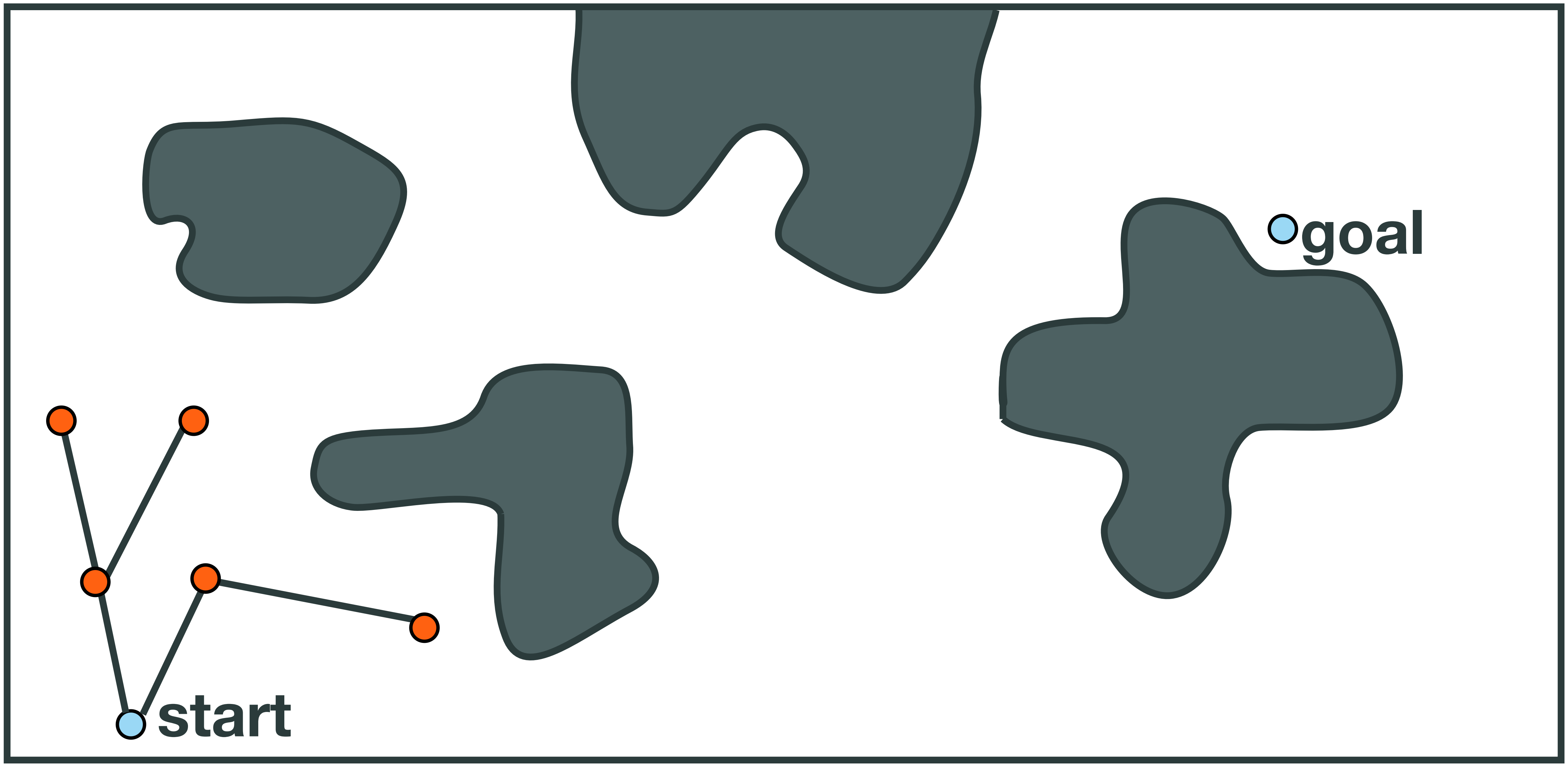
A generic sampling-based tree planner

Sampling-based tree planner operation

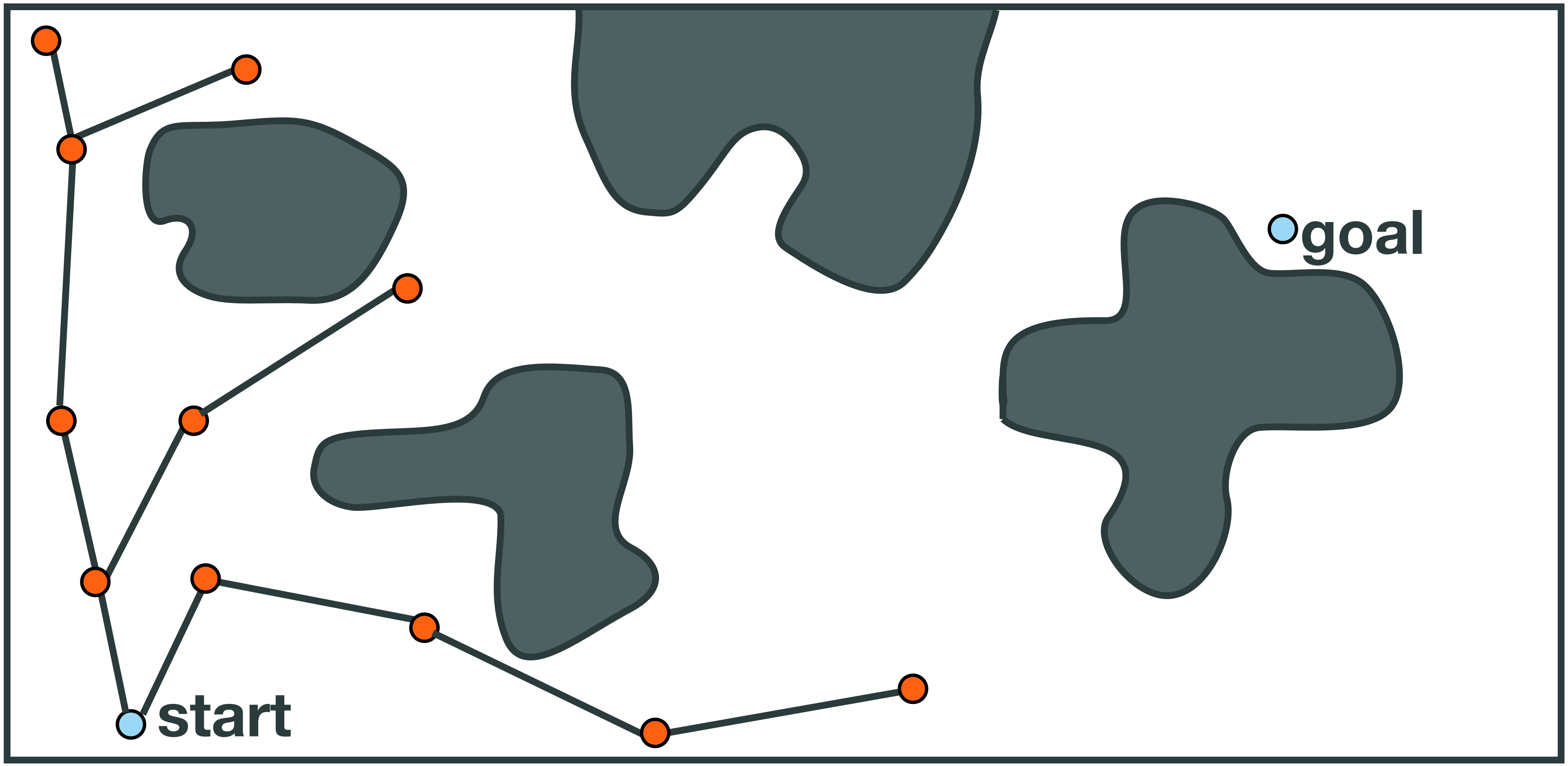


Sampling-based tree planner operation



grow random tree from **start**

Sampling-based tree planner operation



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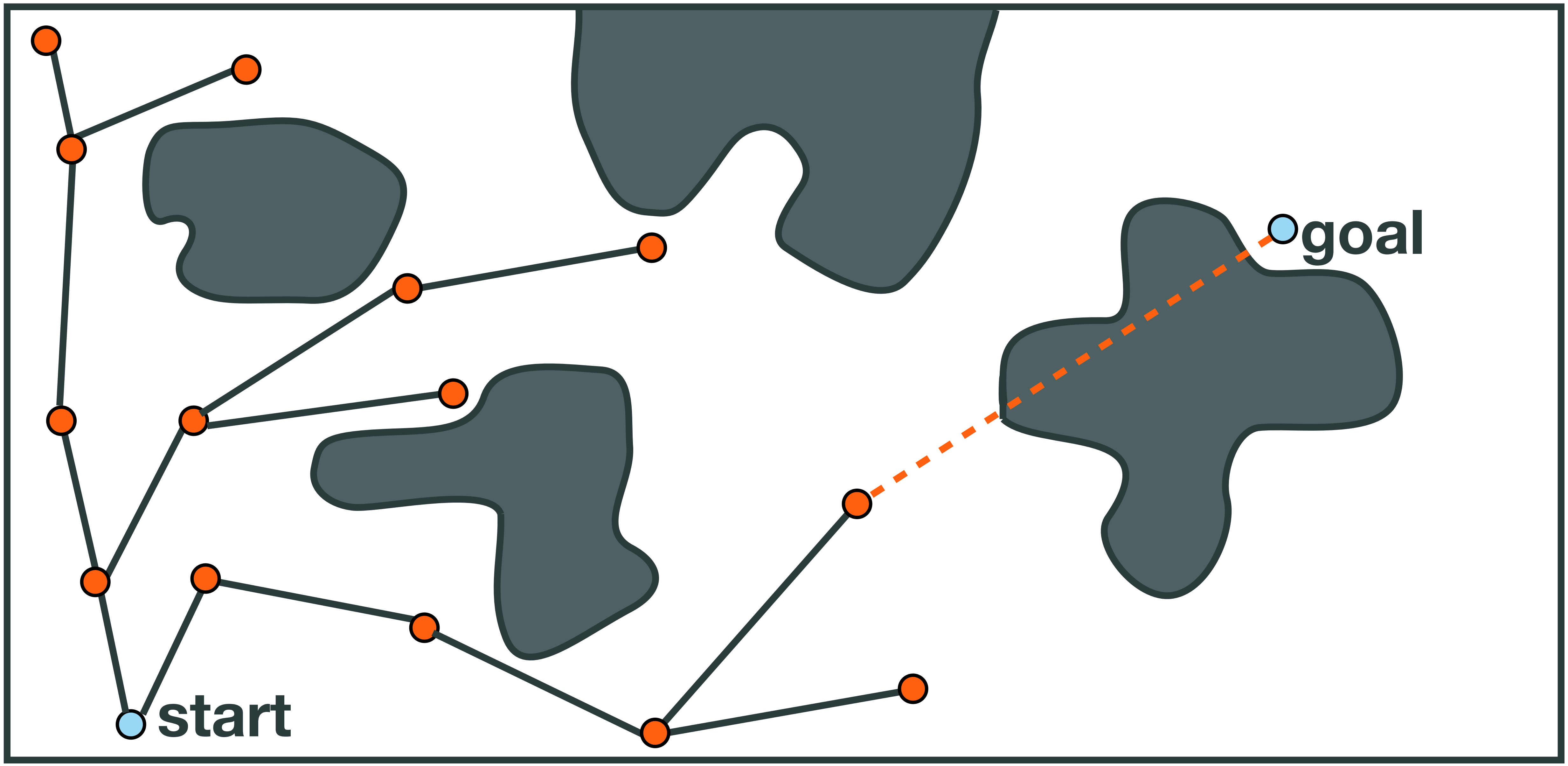
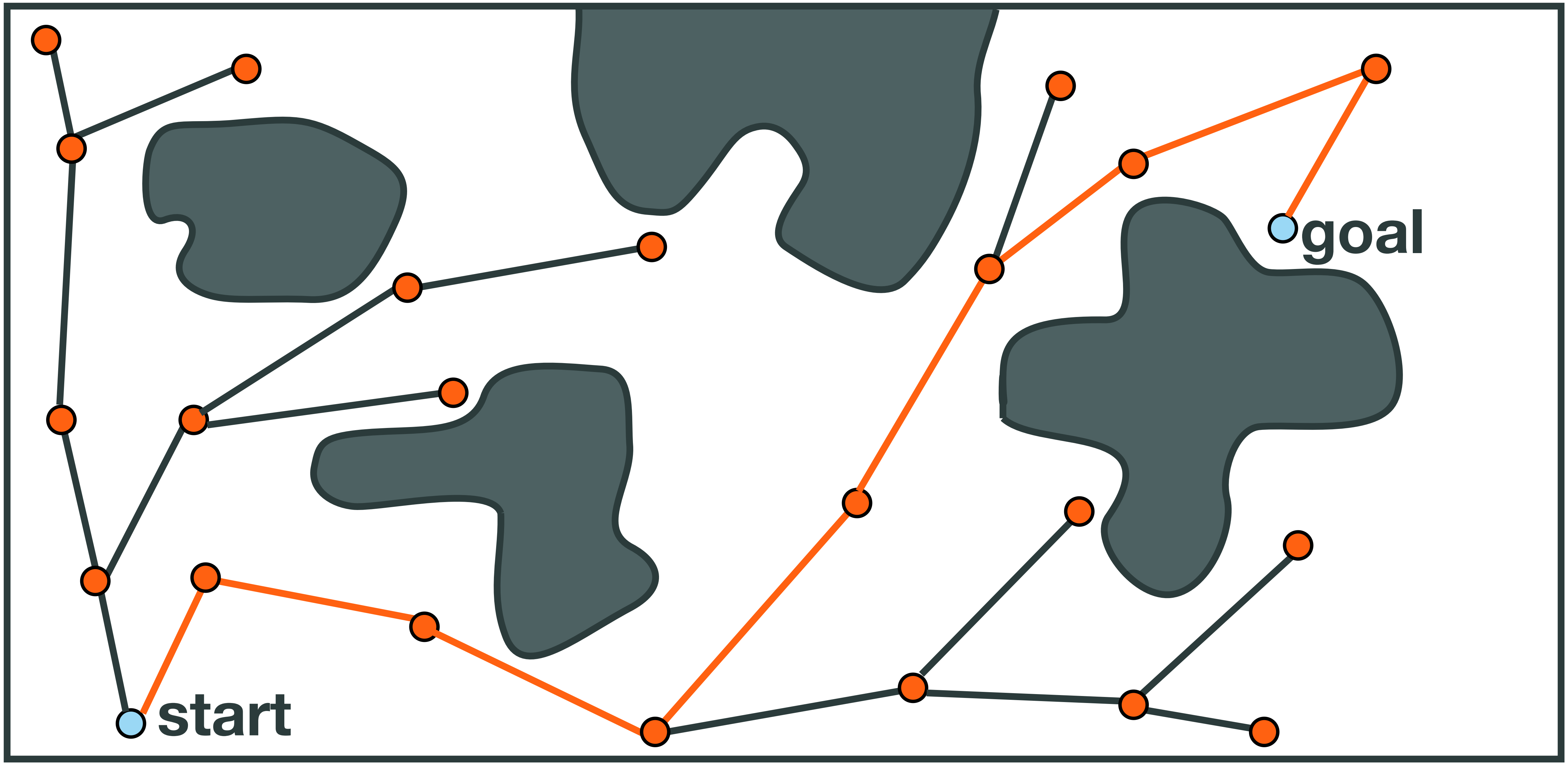


: occasionally attempt to connect tree to **goal**

Sampling-based tree planner operation



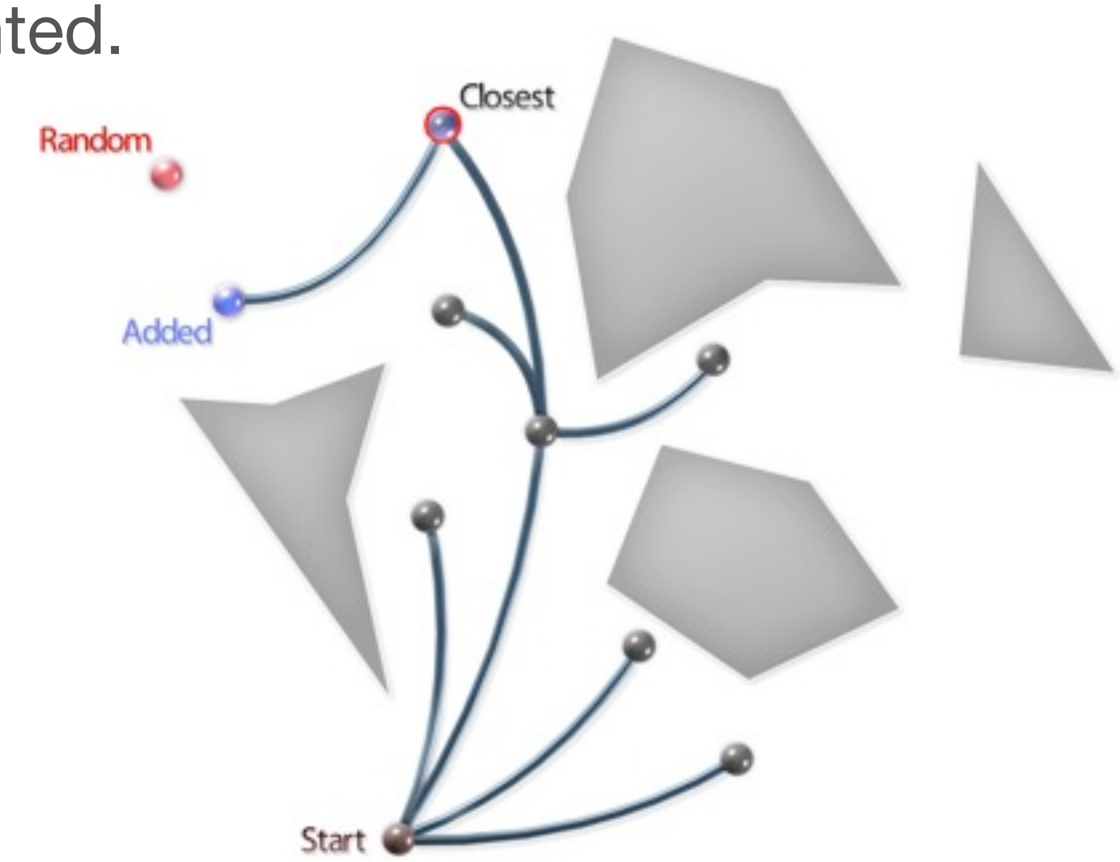
- Repeat until **goal** is connected to tree.
- Bi-directional trees are possible when considering only geometric constraints.

Primitives

- Select Sample
- Expand from the sample
- Store efficiently

Rapidly Exploring Random Trees (RRT)

- Uses proximity query to guide construction (Voronoi Bias).
- Uses propagation instead of connection.
- Powerful heuristic for single-query planning.
- Bi-directional search can be implemented.

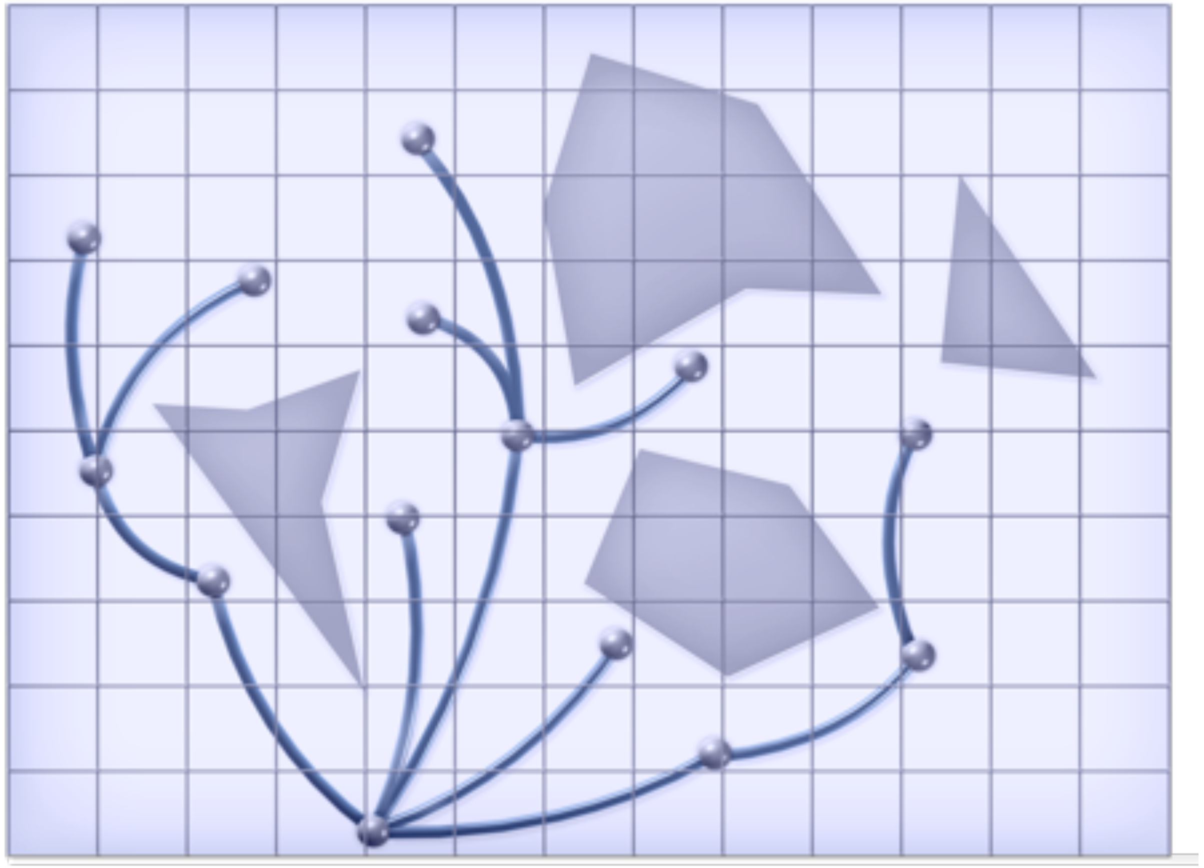


[Lavalle, Kuffer 1999, 2000]

Expansive Trees (EST and SBL)

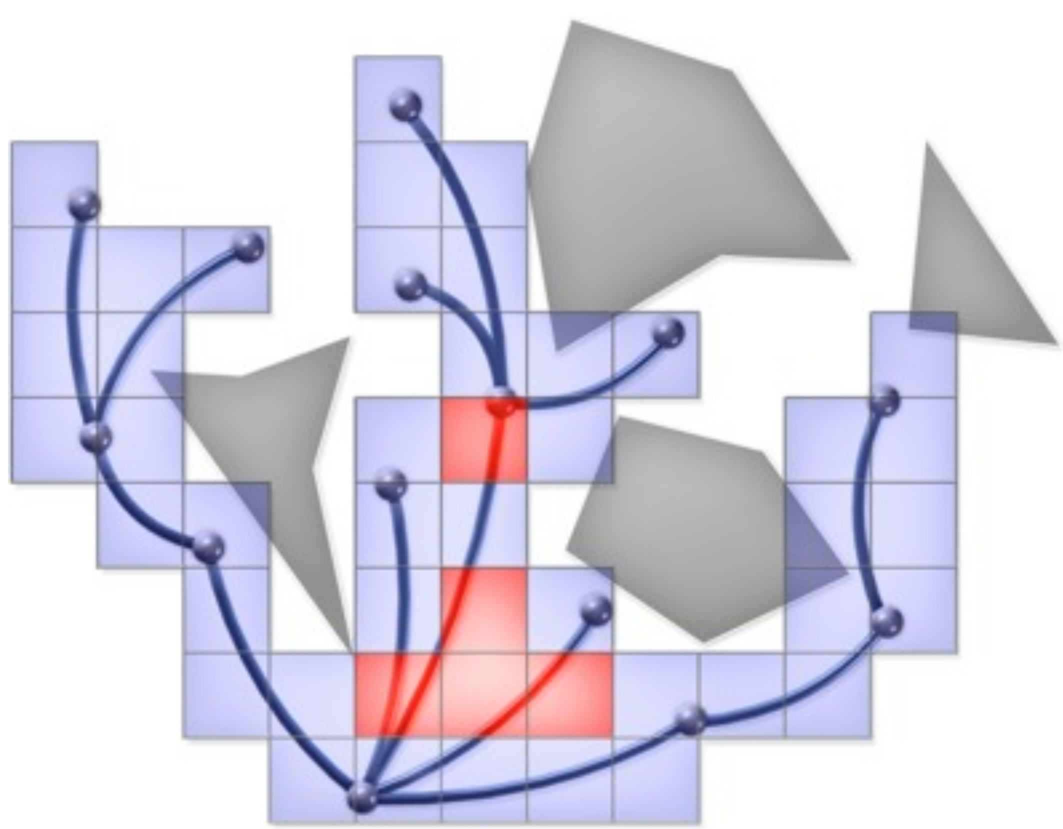
- EST: Uses density of nodes to guide expansion (density bias). [Hsu and Latombe, 1997, 1999]

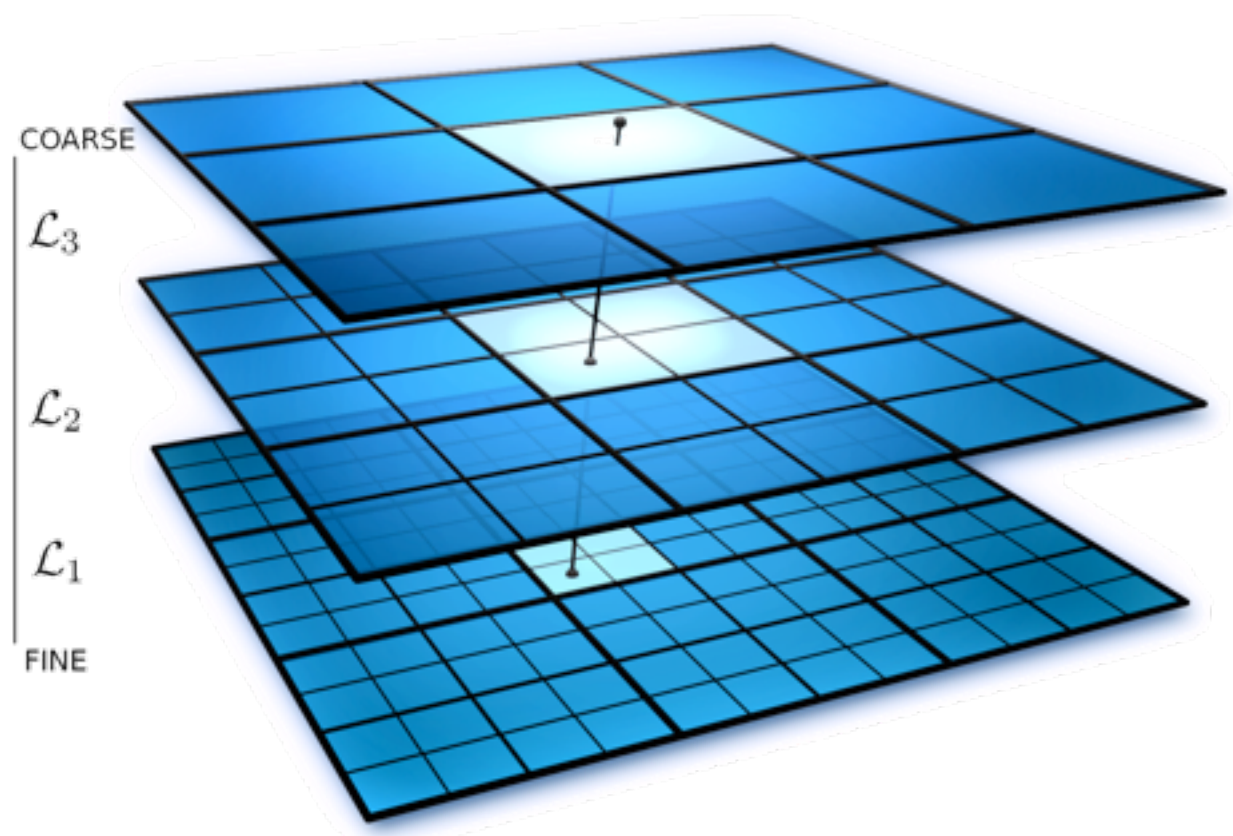
- SBL: Uses some coverage estimates and density of nodes. [Sanchez and Latombe, 2001]



KPIECE

- Keeps tract of coverage by using discretization and by distinguishing the boundary from the covered space.
- Keeping of coverage can be done in a hierarchical fashion.
- Projections my be used.

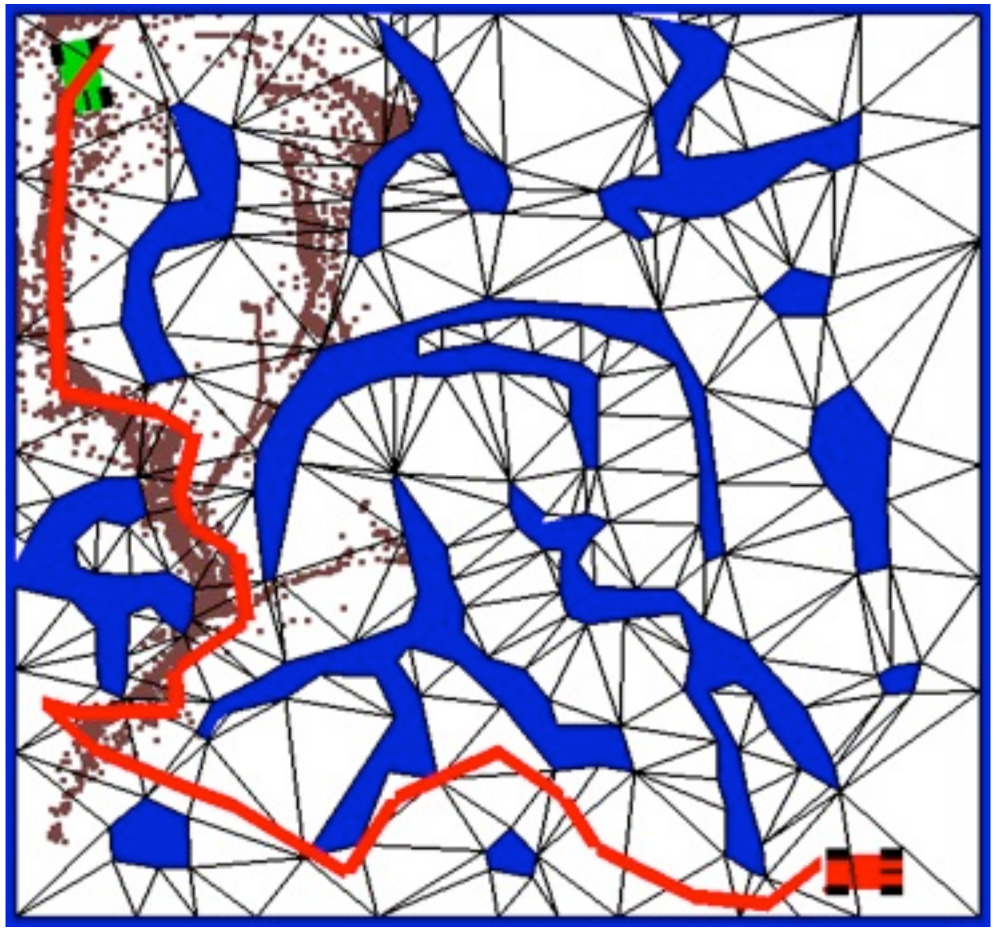




[Șucan, Kavraki 2008]

SyCLoP

- Using a discrete lead to help guide the expansion of the tree



Plaku and Kavraki, 2008

Performance improvements for trees

- Bi-directional search.
- Lazy collision checking.
- Goal biasing.
- Accounting for constrained manifolds.
- Employing motion primitives.
- and many others.

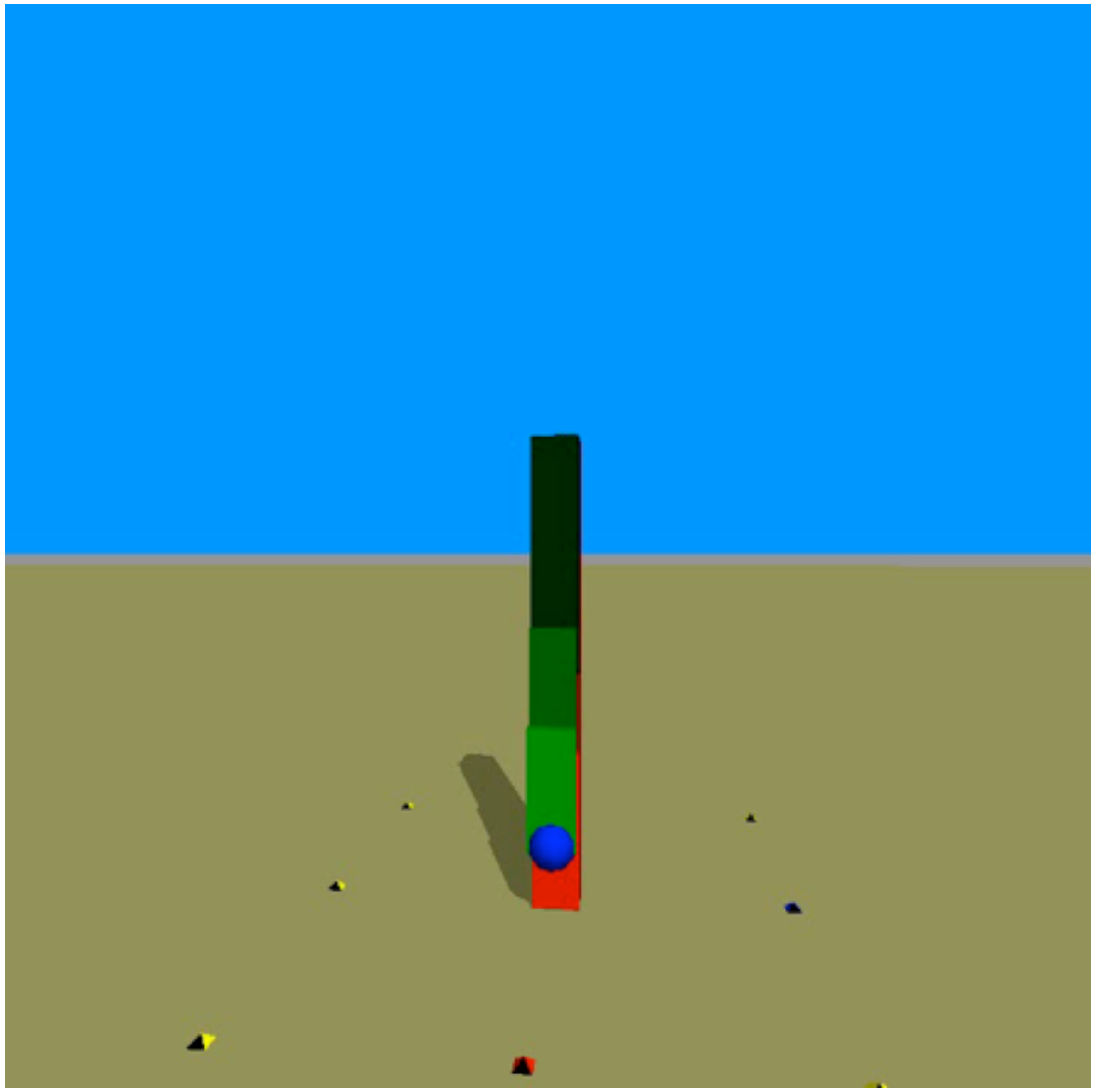
Planning with Dynamics:
Trees offer an advantage

Planning with dynamics

Bekris et al.

Ladd et al.

Planning with dynamics



Ladd et al.

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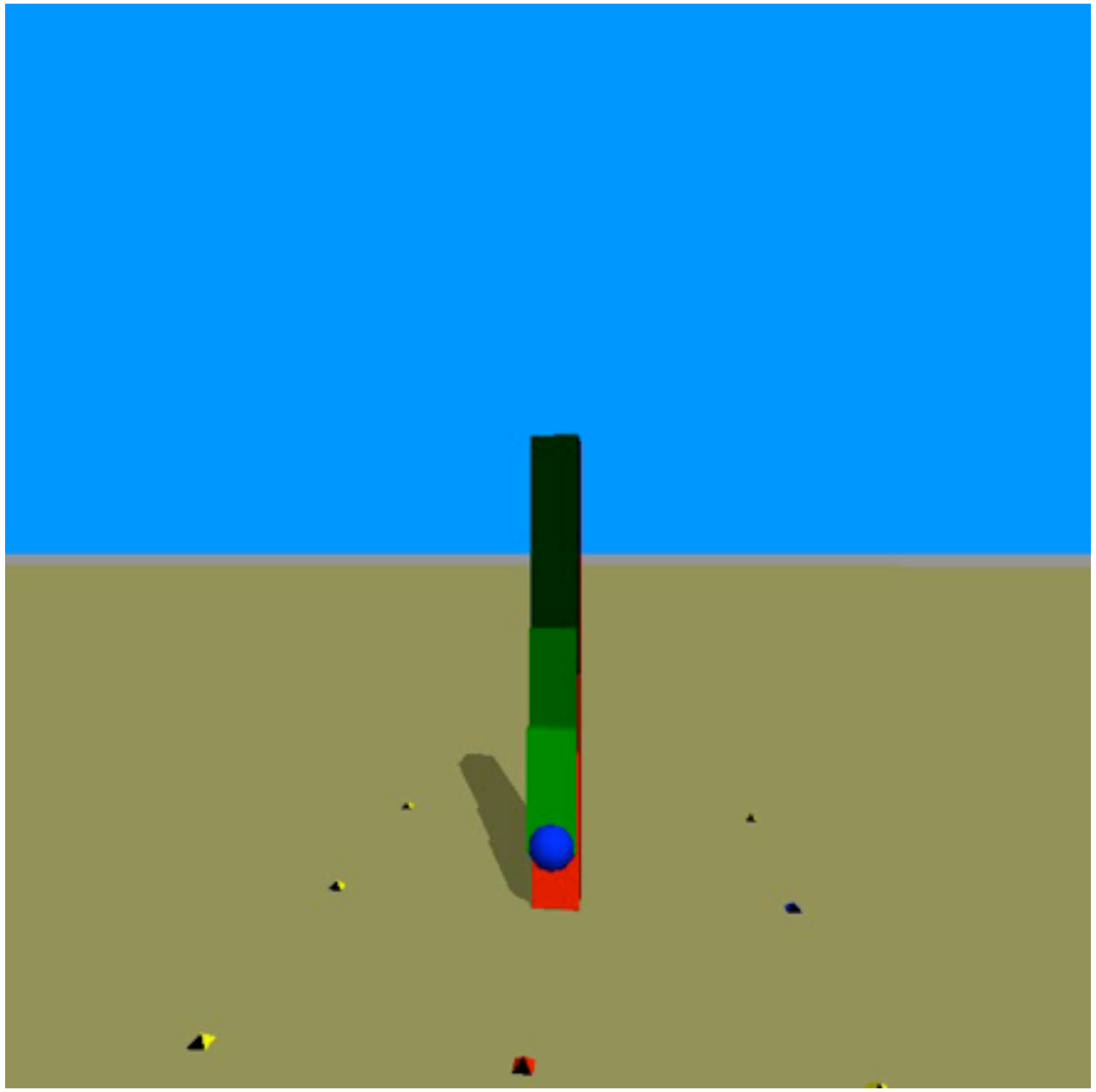

Ladd et al.

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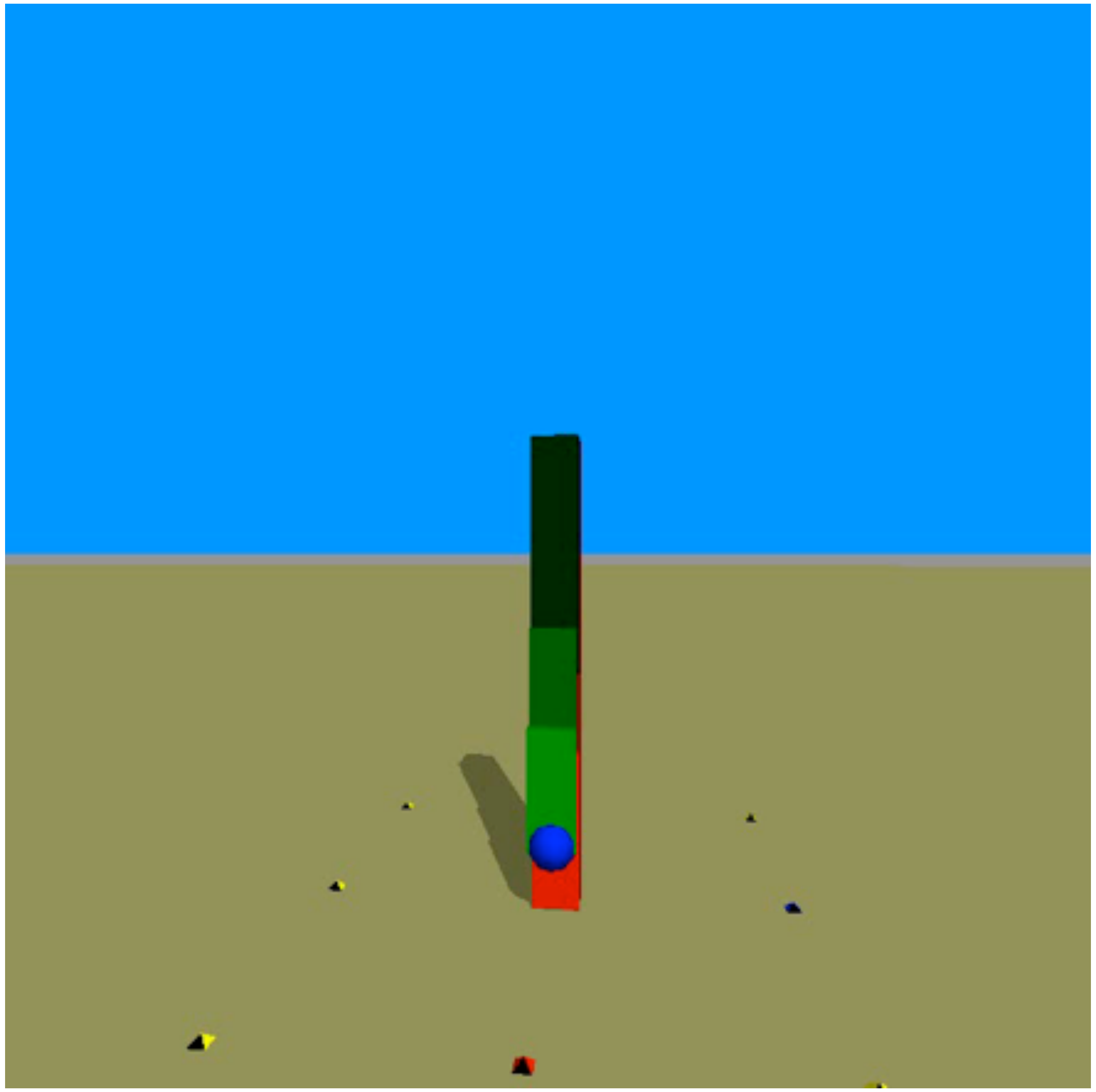
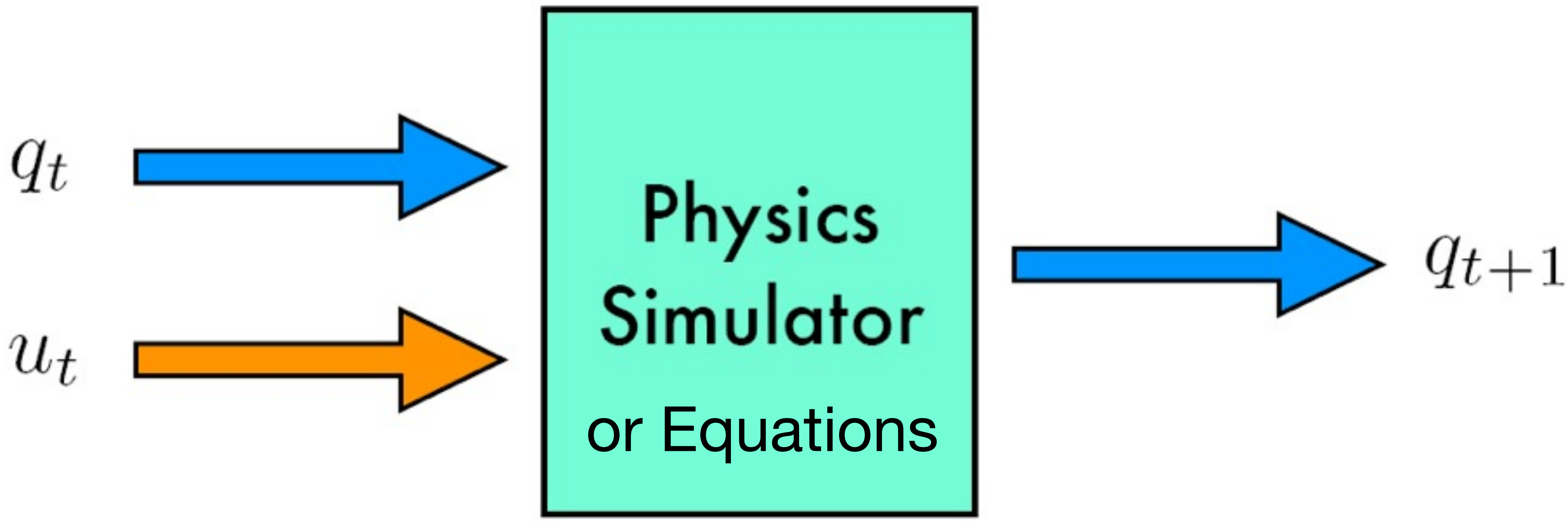


Ladd et al.

Bekris et al.

Physical Systems Planning



$$F(q_t, u_t) = q_{t+1}$$

Space of controls is defined

Physical system planning

Given

1. an initial state $q_0 \in Q$

2. a goal set $G \subset Q$

The discrete physical systems planning problem is to compute a sequence $u_0, \ldots, u_N$ such that:

$$F(q_i, u_i) = q_{i+1}$$

and $q_{N+1} \in G$ is contained in the goal set.

Planning with dynamics

- Adding dynamics is essential to increase physical realism.
- Techniques from control theory can be used to create better paths or reduce differential equation integrations.
- Metrics tend to work poorly.
- Efficient planning for systems with dynamics is still fairly open: sampling-based tree planners offer an advantage.

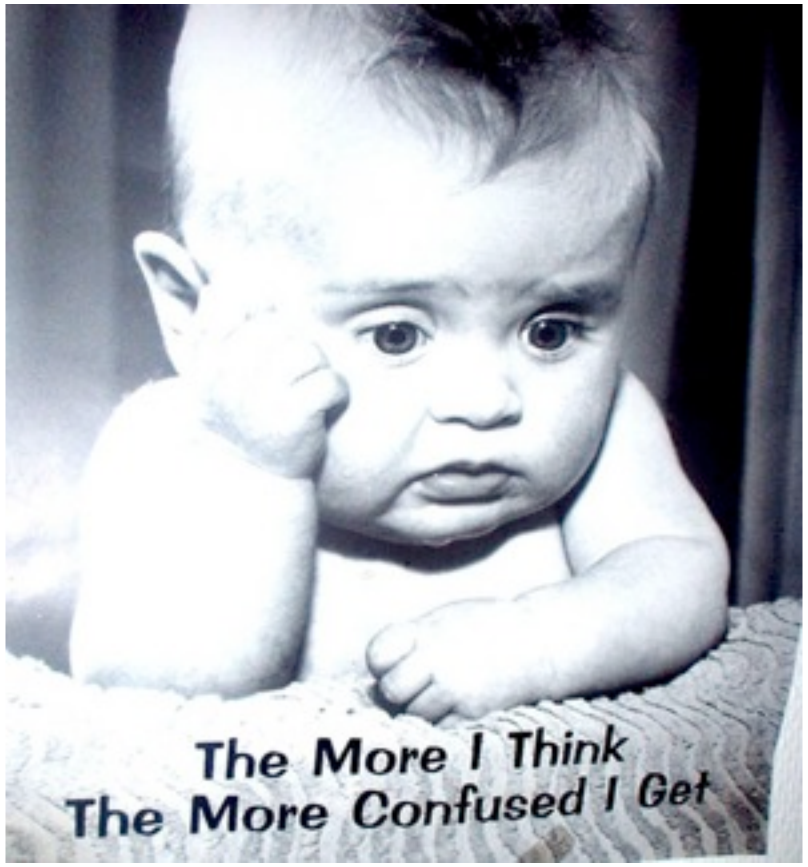
Primitives

- **Select Sample**
- **Expand from the sample**
- **Store efficiently**

These primitives are combined with various optimizations.

Variations of tree sampling-based planners

EST [Hsu et al.’97, ‘00]
RRT [Kuffner, LaValle ’98]
RRT-Connect [Kuffner, LaValle '00]
SBL [Sanchez, Latombe ’01]
Guided EST [Phillips et al. ‘03]
PDRRT [Ranganathan, Koenig '04]
SRT [Plaku et al. '05]
DDRRT [Yershova et al. ’05]
ADDRRT [Jaillet et al. ’05]
RRT-Blossom [Kalisiak, van Panne ’06]
PDST [Ladd, Kavraki ‘06]
Utility RRT [Burns, Brock ’07]
GRIP [Bekris, Kavraki ’07]
Multiparticle RRT [Zucker et al. ’07]
TC-RRT [Stillman et al. '07]
RRT-JT [Vande Wege et al '07]
DSLX [Plaku, Kavraki, Vardi '08]
KPIECE [Șucan, Kavraki '08]
RPDST [Tsianos, Kavraki '08]
BiSpace [Diankov et al. '08]
GRRT [Chakravorty, Kumar '09]
IKBiRRT [Berenson et al.'09]
CBiRRT [Berenson et al.'09]
J+RRT [Vahrenkamp '09]
RRT* [Karaman et al, 10]
and many others



Sampling-based planning (many possibilities)

- Core operations
  - state sampling
  - connection strategy
  - ......
- Common optimizations
  - bi-directional
  - goal-biasing
  - .....

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Need for a systematized approach: OMPL

Benefits

- A repository of planners: choose the right planner and right parameters for that planner.
- Compare new planners to existing ones.
- Develop significantly more complex specialized planners.
- Enable challenging research.
- Support education of new scientists.

Challenges

- Uncertainty.
- Manipulation of rigid and flexible objects.
- Parallel Linkages.
- Dynamics.
- Hybrid planning.
- Real-time planning.
- and other.

THANK YOU

Acknowledgements: Work at the Kavraki Lab on sampling-based planners has been supported by NSF